Xu, X.; **Nigmatulin, R. I.**

On linear instability of atmospheric quasi-hydrostatic equations in response to small shortwave perturbations. (English) Zbl 1475.76048

Lobachevskii J. Math. 42, No. 9, 2237-2256 (2021).

Summary: A set of 3-dimensional atmospheric-dynamics equations with quasi-hydrostatic approximation is proposed and justified with the practical goal to optimize atmospheric modelling at scales ranging from meso meteorology to global climate. Sound waves are filtered by applying the quasi-hydrostatic approximation. In the closed system of hydro/thermodynamic equations, the inertial forces are negligibly small compared to gravity forces, and the asymptotically exact equation for vertical velocity is obtained. Investigation of the stability of solutions to this system in response to small shortwave perturbations has shown that solutions have the property of shortwave instability. There are situations when the increment of the perturbation amplitude tends to infinity, corresponding to absolute instability. It means that the Cauchy problem for such equations may be ill-posed. Its formulation can become conditionally correct if solutions are sought in a limited class of sufficiently smooth functions whose Fourier harmonics tend to zero reasonably quickly when the wavelengths of the perturbations approach zero. Thus, the numerical scheme for the quasi-hydrostatic equations using the finite-difference method requires an adequately selected pseudo-viscosity to eliminate the instability caused by perturbations with wavelengths of the order of the grid size. The result is useful for choosing appropriate vertical and horizontal grid sizes for modelling to avoid shortwave instability associated with the property of the system of equations. Implementation of pseudo-viscosities helps to smoothen or suppress the perturbations that occur during modelling.

MSC:

76E20 Stability and instability of geophysical and astrophysical flows
76U60 Geophysical flows
76M20 Finite difference methods applied to problems in fluid mechanics
86A10 Meteorology and atmospheric physics

Keywords:

3D quasi-hydrostatic atmospheric modelling; linear instability; shortwave perturbation; pseudo-viscosity effect; finite difference scheme

Full Text: DOI

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