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An explicit solution for inelastic buckling of rectangular plates subjected to combined biaxial and shear loads. (English) Zbl 1475.74041

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Summary: In this study, the inelastic buckling equation of a thin plate subjected to all in-plane loads is analytically solved and the inelastic buckling coefficient is explicitly estimated. Using the deformation theory of plasticity, a multiaxial nonlinear stress-strain curve is supposed which is described by the Ramberg-Osgood representation and the von Mises criterion. Due to buckling, the variations are applied on the secant modulus, the Poisson's ratio and the normal and shear strains. Then, the inelastic buckling equation of a perfect thin rectangular plate subjected to combined biaxial and shear loads is completely developed. Applying the generalized integral transform technique, the equation is straightforwardly converted to an eigenvalue problem in a dimensionless form. Initially, a geometrical solution and an algorithm are presented to find the lowest inelastic buckling coefficient (k_s). The solution is successfully validated by some results in the literature. Then, a semi-analytical solution is proposed to simplify the calculation of k_s. The method of linear least squares is applied in two stages on the obtained results and an approximate polynomial equation is found which is usually solved by trial and error. The obtained results show good agreement between the proposed semi-analytical and geometrical methods, so that the differences are $< 12\%$. The semi-analytical solution is easily programmed in usual scientific calculators and can be applied for practical purposes.

MSC:

74G60 Bifurcation and buckling
74K20 Plates
74C05 Small-strain, rate-independent theories of plasticity (including rigid-plastic and elasto-plastic materials)
74G10 Analytic approximation of solutions (perturbation methods, asymptotic methods, series, etc.) of equilibrium problems in solid mechanics

Keywords:

in-plane loading; von Mises criterion; multiaxial stress-strain curve; integral transform technique; semi-analytical solution

Software:

Python

Full Text: DOI

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