

**Xiang, Jianli; Yan, Guozheng**

**The uniqueness of the inverse transmission problem with phaseless far field data at a fixed frequency.** (English) [Zbl 1475.35418](#)

*J. Math. Anal. Appl.* 506, No. 2, Article ID 125691, 18 p. (2022).

**Summary:** In this paper, we establish the unique determination results for inverse acoustic scattering of a penetrable anisotropic obstacle by using phaseless far field data at a fixed frequency. It is well-known that the modulus of the far field pattern is invariant under translations of the scattering obstacle if only one plane wave is used as the incident field, so it is impossible to reconstruct the location of the underlying scatterers. Based on some new research results on the impenetrable obstacle and inhomogeneous isotropic medium, we develop four methods to break the translation invariance property. In the first part, we obtain two uniqueness results by the superposition of two plane waves as the incident field. Then we establish another two uniqueness results by taking the superposition of a plane wave and point sources with different scattering strengths as the incident field.

**MSC:**

[35R30](#) Inverse problems for PDEs

[35J57](#) Boundary value problems for second-order elliptic systems

[35P25](#) Scattering theory for PDEs

[78A45](#) Diffraction, scattering

**Keywords:**

[inverse scattering problem](#); [uniqueness](#); [phaseless far field data](#); [factorization method](#); [reference ball](#)

**Full Text:** [DOI](#)

**References:**

- [1] Brezis, H., *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, Universitext (2011), Springer: Springer New York · [Zbl 1220.46002](#)
- [2] Cakoni, F.; Colton, D., *A Qualitative Approach to Inverse Scattering Theory* (2014), Springer: Springer Berlin · [Zbl 1302.35001](#)
- [3] Cakoni, F.; Colton, D.; Haddar, H., *Inverse scattering theory and transmission eigenvalues*, (*Inverse Scattering Theory* (2016)) · [Zbl 1366.35001](#)
- [4] Cakoni, F.; Colton, D.; Haddar, H., The linear sampling method for anisotropic media, *J. Comput. Appl. Math.*, 146, 285-299 (2001) · [Zbl 1019.78003](#)
- [5] Colton, D.; Kress, R., *Inverse Acoustic and Electromagnetic Scattering Theory* (2019), Springer: Springer Berlin · [Zbl 1425.35001](#)
- [6] Colton, D.; Kress, R., Eigenvalues of the far field operator for the Helmholtz equation in an absorbing medium, *SIAM J. Appl. Math.*, 55, 1724-1735 (1995) · [Zbl 0839.35094](#)
- [7] Colton, D.; Kress, R., Eigenvalues of the far field operator and inverse scattering theory, *SIAM J. Math. Anal.*, 26, 601-615 (1995) · [Zbl 0826.35134](#)
- [8] Dong, H.; Lai, J.; Li, P. J., Inverse obstacle scattering problem for elastic waves with phased or phaseless far-field data, *SIAM J. Imaging Sci.*, 12, 809-838 (2019)
- [9] Dong, H.; Lai, J.; Li, P. J., An inverse acoustic-elastic interaction problem with phased or phaseless far-field data, *Inverse Probl.*, 36, Article 035014 pp. (2020) · [Zbl 1437.35702](#)
- [10] Hähner, P., On the uniqueness of the shape of a penetrable, anisotropic obstacle, *J. Comput. Appl. Math.*, 116, 167-180 (2000) · [Zbl 0978.35098](#)
- [11] Ivanyshyn, O., Shape reconstruction of acoustic obstacles from the modulus of the far field pattern, *Inverse Probl. Imaging*, 1, 609-622 (2007) · [Zbl 1194.35502](#)
- [12] Ivanyshyn, O.; Kress, R., Identification of sound-soft 3D obstacles from phaseless data, *Inverse Probl. Imaging*, 4, 131-149 (2010) · [Zbl 1220.35194](#)
- [13] Ji, X.; Jia, Y. F.; Liu, X. D., Inverse fluid-solid interaction scattering problem using phased and phaseless far field data, *Acta Math. Appl. Sin. Engl. Ser.*, 36, 74-94 (2020) · [Zbl 1431.35245](#)
- [14] Ji, X.; Liu, X. D., Inverse elastic scattering problems with phaseless far field data, *Inverse Probl.*, 35, Article 114004 pp.

(2019) · [Zbl 1423.35449](#)

- [15] Ji, X.; Liu, X. D., Inverse electromagnetic source scattering problems with multi-frequency sparse phased and phaseless far field data, *SIAM J. Sci. Comput.*, 41, B1368-B1388 (2019) · [Zbl 1428.35680](#)
- [16] Ji, X.; Liu, X. D.; Zhang, B., Target reconstruction with a reference point scatterer using phaseless far field patterns, *SIAM J. Imaging Sci.*, 12, 372-391 (2019) · [Zbl 1423.35286](#)
- [17] Ji, X.; Liu, X. D.; Zhang, B., Phaseless inverse source scattering problem: phase retrieval, uniqueness and direct sampling methods, *J. Comput. Phys.* X, 1, Article 100003 pp. (2019)
- [18] Ji, X.; Liu, X. D.; Zhang, B., Inverse acoustic scattering with phaseless far field data: uniqueness, phase retrieval, and direct sampling methods, *SIAM J. Imaging Sci.*, 12, 1163-1189 (2019)
- [19] Karageorghis, A.; Johansson, B. T.; Lesnic, D., The method of fundamental solutions for the identification of a sound-soft obstacle in inverse acoustic scattering, *Appl. Numer. Math.*, 62, 1767-1780 (2012) · [Zbl 1255.65203](#)
- [20] Kirsch, A.; Grinberg, N., *The Factorization Method for Inverse Problems* (2008), Oxford University Press: Oxford University Press Oxford · [Zbl 1222.35001](#)
- [21] Kirsch, A.; Liu, X. D., The factorization method for inverse acoustic scattering by a penetrable anisotropic obstacle, *Math. Methods Appl. Sci.*, 37, 1159-1170 (2014) · [Zbl 1290.35331](#)
- [22] Kirsch, A.; Liu, X. D., A modification of the factorization method for the classical acoustic inverse scattering problems, *Inverse Probl.*, 30, Article 035013 pp. (2014) · [Zbl 1291.35223](#)
- [23] Kress, R.; Rundell, W., Inverse obstacle scattering with modulus of the far field pattern as data, (*Inverse Problems in Medical Imaging and Nondestructive Testing* (1997)), 75-92 · [Zbl 0880.65105](#)
- [24] Kwon, O.; Seo, J. K., Lipschitz stability estimates for translations and balls in inverse scattering, *Inverse Probl.*, 16, 293-301 (2000) · [Zbl 0974.35135](#)
- [25] Lee, K. M., Shape reconstructions from phaseless data, *Eng. Anal. Bound. Elem.*, 71, 174-178 (2016) · [Zbl 1403.76158](#)
- [26] Li, J. Z.; Liu, H. Y.; Zou, J., Strengthened linear sampling method with a reference ball, *SIAM J. Sci. Comput.*, 31, 4013-4040 (2009) · [Zbl 1205.35339](#)
- [27] Liu, J.; Seo, J., On stability for a translated obstacle with impedance boundary condition, *Nonlinear Anal.*, 59, 731-744 (2004) · [Zbl 1062.35175](#)
- [28] Liu, X. D.; Zhang, B., Unique determination of a sound soft ball by the modulus of a single far field datum, *J. Math. Anal. Appl.*, 365, 619-624 (2010) · [Zbl 1185.35329](#)
- [29] Majda, A., High frequency asymptotics for the scattering matrix and the inverse problem of acoustical scattering, *Commun. Pure Appl. Math.*, 29, 261-291 (1976) · [Zbl 0463.35048](#)
- [30] Qin, H. H.; Liu, X. D., The interior inverse scattering problem for cavities with an artificial obstacle, *Appl. Numer. Math.*, 88, 18-30 (2015) · [Zbl 1309.76163](#)
- [31] Shin, J., Inverse obstacle backscattering problems with phaseless data, *Eur. J. Appl. Math.*, 27, 111-130 (2016) · [Zbl 1383.78023](#)
- [32] Xu, X. X.; Zhang, B.; Zhang, H. W., Uniqueness in inverse scattering problems with phaseless far-field data at a fixed frequency, *SIAM J. Appl. Math.*, 78, 1737-1753 (2018) · [Zbl 1394.78015](#)
- [33] Xu, X. X.; Zhang, B.; Zhang, H. W., Uniqueness in inverse scattering problems with phaseless far-field data at a fixed frequency II, *SIAM J. Appl. Math.*, 78, 3024-3039 (2018) · [Zbl 1403.78010](#)
- [34] Xu, X. X.; Zhang, B.; Zhang, H. W., Uniqueness in inverse acoustic and electromagnetic scattering with phaseless near-field data at a fixed frequency, *Inverse Probl. Imaging*, 14, 489-510 (2020) · [Zbl 07208030](#)
- [35] Xu, X. X.; Zhang, B.; Zhang, H. W., Uniqueness in inverse electromagnetic scattering problem with phaseless far-field data at a fixed frequency, *IMA J. Appl. Math.*, 85, 823-839 (2020) · [Zbl 1465.78004](#)
- [36] Zhang, B.; Zhang, H. W., Recovering scattering obstacles by multi-frequency phaseless far-field data, *J. Comput. Phys.*, 345, 58-73 (2017) · [Zbl 1378.35210](#)
- [37] Zhang, B.; Zhang, H. W., Fast imaging of scattering obstacles from phaseless far-field measurements at a fixed frequency, *Inverse Probl.*, 34, Article 104005 pp. (2018) · [Zbl 1452.65311](#)
- [38] Zhang, B.; Zhang, H. W., Imaging of locally rough surfaces from intensity-only far-field or near-field data, *Inverse Probl.*, 33, Article 055001 pp. (2017) · [Zbl 1388.35227](#)
- [39] Zhang, D. Y.; Guo, Y. K., Uniqueness results on phaseless inverse scattering with a reference ball, *Inverse Probl.*, 34, Article 085002 pp. (2018) · [Zbl 1442.35553](#)
- [40] Zhang, D. Y.; Guo, Y. K.; Sun, F.; Liu, H., Unique determinations in inverse scattering problems with phaseless near-field measurements, *Inverse Probl. Imaging*, 14, 569-582 (2020) · [Zbl 1441.78017](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.