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Conditional stability for an inverse coefficient problem of a weakly coupled time-fractional diffusion system with half order by Carleman estimate. (English) Zbl 1475.35397

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Summary: Under a priori boundedness conditions of solutions and coefficients, we prove a Hölder stability estimate for an inverse problem of determining two spatially varying zeroth order non-diagonal elements of a coefficient matrix in a one-dimensional fractional diffusion system of half order in time. The proof relies on the conversion of the fractional diffusion system to a system of order 4 in the space variable and the Carleman estimate.

MSC:

35R11 Fractional partial differential equations

35R30 Inverse problems for PDEs

35K51 Initial-boundary value problems for second-order parabolic systems

Keywords:

fractional diffusion equation; inverse coefficient problem; Carleman estimate; conditional stability; Caputo derivative

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