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A survey on asymptotic stability of ground states of nonlinear Schrödinger equations. II.

(English) [Zbl 1475.35313](#)

Discrete Contin. Dyn. Syst., Ser. S 14, No. 5, 1693-1716 (2021).

Summary: We give short survey on the question of asymptotic stability of ground states of nonlinear Schrödinger equations, focusing primarily on the so called nonlinear Fermi Golden Rule.

For Part I, see [the first author, Quad. Mat. 15, 21–57 (2004; [Zbl 1130.35360](#))].

MSC:

[35Q55](#) NLS equations (nonlinear Schrödinger equations)

[35B40](#) Asymptotic behavior of solutions to PDEs

Cited in **2** Documents

Keywords:

ground state; stability; dispersion; normal form; metastability

Full Text: [DOI](#) [arXiv](#)

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