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New critical exponents for a doubly singular parabolic equation. (English) Zbl 1475.35188

Appl. Anal. 100, No. 11, 2386-2404 (2021).

Summary: This paper deals with the Cauchy problem for a doubly singular parabolic equation with nonlocal inner source

$$u_t = \operatorname{div}(|\nabla u^m|^{p-2}\nabla u^m) + \|u\|^{r-1}_{L^q(\mathbb{R}^N)}u^{s+1}, \quad (x,t) \in \mathbb{R}^N \times (0,T),$$

where $N \geq 1, 1-((1+m)/(1+mN)) < m(p-1) < 1, 0 < m \leq 1, q > 1, r \geq 1, 0 \leq s < p/N+m(p-1)-1$ and $r+s > 1$. We first obtain a new critical Fujita exponent by virtue of the auxiliary function method and the forward self-similar solution, and then determine the second critical exponent to classify global and non-global solutions of the problem in the coexistence region via the decay rates of an initial data at spatial infinity. Moreover, the large time behavior of global solution and the life span of non-global solution are derived.

MSC:

35K67 Singular parabolic equations
35B33 Critical exponents in context of PDEs
35B40 Asymptotic behavior of solutions to PDEs
35B44 Blow-up in context of PDEs
35K15 Initial value problems for second-order parabolic equations
35K59 Quasilinear parabolic equations
35K65 Degenerate parabolic equations
35R09 Integro-partial differential equations

Keywords:

doubly singular parabolic equation; nonlocal inner source

Full Text: DOI

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