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Local null-controllability of a nonlocal semilinear heat equation. (English) Zbl 1475.35184

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Authors' abstract: This paper deals with the problem of internal null-controllability of a heat equation posed on a bounded domain with Dirichlet boundary conditions and perturbed by a semilinear nonlocal term. We prove the small-time local null-controllability of the equation. The proof relies on two main arguments. First, we establish the small time local null-controllability of a 2×2 reaction-diffusion system, where the second equation is governed by the parabolic operator $\tau\partial_t - \sigma\Delta, \tau, \sigma > 0$. More precisely, this controllability result is obtained uniformly with respect to the parameters $(\tau, \sigma) \in (0, 1) \times (1, +\infty)$. Secondly, we observe that the semilinear nonlocal heat equation is actually the asymptotic derivation of the reaction-diffusion system in the limit $(\tau, \sigma) \to (0, +\infty)$. Finally, we illustrate these results by numerical simulations.

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MSC:

35K58 Semilinear parabolic equations
93B05 Controllability
93B07 Observability
93C20 Control/observation systems governed by partial differential equations
35K20 Initial-boundary value problems for second-order parabolic equations

Keywords:

nonlocal heat equation; local null-controllability; semilinear system; Carleman inequalities

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