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**Second order necessary conditions for optimal control problems of evolution equations involving final point equality constraints.** (English) [Zbl 1473.49028](#)

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Summary: We establish some second order necessary conditions for optimal control problems of evolution equations involving final point equality and inequality constraints. Compared with the existing works, the main difference is due to the presence of end-point equality constraints. With such constraints, we cannot simply use the variational techniques since perturbations of a given control may be no longer admissible. We also cannot use the Ekeland's variational principle, which is a first order variational principle, to obtain second order necessary conditions. Instead, we combine some inverse mapping theorems on metric spaces and second order linearization of data to obtain our results.

**MSC:**

**49K20** Optimality conditions for problems involving partial differential equations

**Keywords:**

optimal control; time evolution partial differential equation; second order necessary condition; local minimizer

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**References:**

- [1] J.-P. Aubin and H. Frankowska, Set-valued analysis. Springer Science & Business Media (2009). · [Zbl 1168.49014](#)
- [2] P. Cannarsa, G. Da Prato and H. Frankowska, Domain invariance for local solutions of semilinear evolution equations in Hilbert spaces. J. London Math. Soc. 102 (2020) 287-318. · [Zbl 1454.58010](#)
- [3] E. Casas and M. Mateos, Second order optimality conditions for semilinear elliptic control problems with finitely many state constraints. SIAM J. Control Optim. 40 (2002) 1431-1454. · [Zbl 1037.49024](#)
- [4] E. Casas and F. Tr oltzsch, Second-order necessary and sufficient optimality conditions for optimization problems and applications to control theory. SIAM J. Control Optim. 13 (2002) 406-431. · [Zbl 1052.49022](#)
- [5] E. Casas and F. Tr oltzsch, Second order optimality conditions and their role in PDE control. Jahresber. Dtsch. Math.-Ver. 117 (2015) 3-44. · [Zbl 1311.49002](#)
- [6] L.C. Evans, Partial differential equations. American Mathematical Society, Providence, RI (2010). · [Zbl 1194.35001](#)
- [7] H. Frankowska, High order inverse function theorems. Ann. Inst. H. Poincar e Anal. Non Lin aire 6 (1989) 283-303. · [Zbl 0701.49040](#)
- [8] H. Frankowska, On second-order necessary conditions in optimal control of problems with mixed final point constraints. In Proceedings of 58th IEEE Conference on Decision and Control, Nice, France, December 11-13 (2019), DOI: 10.1109/CDC40024.2019.9029546.
- [9] H. Frankowska, D. Hoehener and D. Tonon, A second-order maximum principle in optimal control under state constraints. Serdica Math. J. 39 (2013) 233-270. · [Zbl 1324.49017](#)
- [10] H. Frankowska and N.P. Osmolovskii, Second-order necessary conditions for a strong local minimum in a control problem with general control constraints. Appl. Math. Optim. 80 (2019) 135-164. · [Zbl 1421.49019](#)
- [11] H. Frankowska and N.P. Osmolovskii, Distance estimates to feasible controls for systems with final point constraints and second order necessary optimality conditions. Syst. Control Lett. 144 (2020) 104770. · [Zbl 1454.49008](#)
- [12] H. Frankowska, H. Zhang and X. Zhang, Necessary optimality conditions for local minimizers of stochastic optimal control problems with state constraints. Trans. Amer. Math. Soc. 372 (2019) 1289-1331. · [Zbl 1417.93335](#)
- [13] X. Li and J. Yong, Optimal control theory for infinite dimensional systems. Birkh user Boston, Inc., Boston, MA (1995).
- [14] X. Liu, Q. L u and X. Zhang, Finite codimensional controllability, and optimal control problems with endpoint state constraints. J. Math. Pures Appl. 138 (2020) 164-203. · [Zbl 1441.93033](#)
- [15] Q. L u and X. Zhang, General Pontryagin-type stochastic maximum principle and backward stochastic evolution equations in infinite dimensions. Springer (2014). · [Zbl 1316.49004](#)
- [16] A. Pazy, Semigroups of linear operators and applications to partial differential equations. Springer-Verlag, New York (1983). · [Zbl 0516.47023](#)
- [17] L.S. Pontryagin, V.G. Boltyanskii, R.V. Gamkrelidze and E.F. Mishchenko, Mathematical theory of optimal processes. Inter-

science Publishers John Wiley & Sons, Inc., New York-London (1962). · [Zbl 0102.32001](#)

- [18] J.-P. Raymond and F. Tröltzsch, Second order sufficient optimality conditions for nonlinear parabolic control problems with state constraints. *Discrete Contin. Dynam. Syst.* 6 (2000) 431-450. · [Zbl 1010.49015](#)
- [19] A. Rösch and F. Tröltzsch, Sufficient second-order optimality conditions for a parabolic optimal control problem with pointwise control-state constraints. *SIAM J. Control Optim.* 42 (2003) 138-154. · [Zbl 1038.49028](#)
- [20] F. Tröltzsch and D. Wachsmuth, Second-order sufficient optimality conditions for the optimal control of Navier-Stokes equations. *ESAIM: COCV* 12 (2006) 93-119. · [Zbl 1111.49017](#) · doi:<https://www.esaim-cocv.org/articles/cocv/abs/2006/01/cocv0431/cocv0431.html>
- [21] J. Yong and X. Zhou, *Stochastic controls. Hamiltonian systems and HJB equations*. Springer-Verlag, New York (1999). · [Zbl 0943.93002](#)
- [22] E. Zuazua, Controllability and observability of partial differential equations: some results and open problems. Vol. 3 of *Handbook of Differential Equations: Evolutionary Equations*. Elsevier Science (2006) 527-621. · [Zbl 1193.35234](#)

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