Garroni, Adriana; **Marziani, Roberta**; **Scala, Riccardo**

Derivation of a line-tension model for dislocations from a nonlinear three-dimensional energy: the case of quadratic growth. (English) Zbl 1473.49015

SIAM J. Math. Anal. 53, No. 4, 4252-4302 (2021).

For a general introduction in problems related to dislocations, one recommends the book of *P. M. Anderson* et al. [Theory of dislocations. 3rd edition. Cambridge: Cambridge University Press (2017; Zbl 1365.82001)] and an extensive article from [*D. J. Bacon* et al., "Anisotropic continuum theory of elastic defects", Progr. Mater. Sci. 23, 51–262 (1979)]. In the second section of the present paper one introduces elements related to the model of dislocation, the set of admissible dislocation densities, (h, α)-dilute measures and one presents a compactness result related to the dislocation measures and associated fields with equibounded energies. This result is completed with a Γ-convergence result for the energy functional. Proofs of both results can be found in the fifth section of the article. The third section is dedicated to an extensive analyses of a three-dimensional cell problem in a nonlinear framework. One combines some known results for linear case with known techniques developed for the two-dimensional case. An analysis of the asymptotic behavior of the quadratic energy associated to a sequence of dilute dislocations is performed in the fourth section. Notations used throughout the paper are shown in the Appendix A.

Reviewer: Claudia Simionescu-Badea (Wien)

MSC:

49J45 Methods involving semicontinuity and convergence; relaxation
74C05 Small-strain, rate-independent theories of plasticity (including rigid-plastic and elasto-plastic materials)
35Q74 PDEs in connection with mechanics of deformable solids
74B10 Linear elasticity with initial stresses
74N05 Crystals in solids
35B65 Smoothness and regularity of solutions to PDEs

Keywords:

dislocations; Γ-convergence; relaxation; nonlinear elasticity

Full Text: DOI

References:

[1] E. Acerbi, V. Chiadò Piat, G. Dal Maso, and D. Percivale, An extension theorem from connected sets, and homogenization in general periodic domains, Nonlinear Anal., 18 (1992), pp. 481-496. · Zbl 0779.35011

[2] G. Alberti, S. Baldo, and G. Orlandi, Variational convergence for functionals of Ginzburg-Landau type, Indiana Univ. Math. J., 54 (2005), pp. 1411-1472. · Zbl 1160.35013

[3] R. Alicandro, M. Cicalese, and M. Ponsiglione, Variational equivalence between Ginzburg-Landau, \(XY\)-spin systems and screw dislocations energies, Indiana Univ. Math. J., 60 (2011), pp. 171-208. · Zbl 1251.49017

[4] R. Alicandro, L. De Luca, A. Garroni, and M. Ponsiglione, Metastability and dynamics of discrete topological singularities in two dimensions: A \(\Gamma \)-convergence approach, Arch. Ration. Mech. Anal., 214 (2014), pp. 269-330. · Zbl 1305.82013

[5] G. Anzellotti, S. Baldo, and D. Percivale, Dimension reduction in variational problems, asymptotic development in \(\Gamma \)-convergence and thin structures in elasticity, Asymptotic Anal., 9 (1994), pp. 61-100. · Zbl 0811.49020

[6] D. J. Bacon, D. M. Barnett, and R. O. Scattergood, Anisotropic continuum theory of elastic defects, Progr. Mater. Sci., 23 (1979), pp. 51-262.

[7] F. Bethuel, H. Brezis, and F. Hélein, Ginzburg-Landau Vortices, Birkhäuser, Basel, 1994.

[8] F. Bethuel, H. Brezis, and G. Orlandi, Asymptotics for the Ginzburg-Landau equation in arbitrary dimensions, J. Funct. Anal., 186 (2001), pp. 432-520. · Zbl 1077.35047

[9] J. Bourgain and H. Brezis, New estimate for the Laplacian, the div-curl, and related Hodge systems, C. R. Math. Acad. Sci. Paris, 338 (2004), pp. 539-543. · Zbl 1101.35013

[10] S. Cacace and A. Garroni, A multiphase transition model for dislocations with interfacial microstructure, Interfaces Free

Bound., 11 (2009), pp. 291-316. · Zbl 1184.35029

[11] P. Cermelli and G. Leoni, Renormalized energy and forces on dislocations, SIAM J. Math. Anal., 37 (2005), pp. 1131-1160, https://doi.org/10.1137/040621636. · Zbl 1141.74023

[12] O. Conti, A. Garroni, and A. Massaccesi, Modeling of dislocations and relaxation of functionals on 1-currents with discrete multiplicity, Calc. Var. Partial Differential Equations, 54 (2015), pp. 1847-1874. · Zbl 1328.49009

[13] S. Conti, A. Garroni, and S. Müller, Singular kernels, multiscale decomposition of microstructure, and dislocation models, Arch. Ration. Mech. Anal., 199 (2011), pp. 779-819. · Zbl 1251.74006

[14] S. Conti, A. Garroni, and S. Müller, Dislocation microstructures and strain-gradient plasticity with one active slip plane, J. Mech. Phys. Solids, 93 (2016), pp. 240-251.

[15] S. Conti, A. Garroni, and S. Müller, Derivation of Strain-Gradient Plasticity from a Generalized Peierls-Nabarro Model, preprint, https://arxiv.org/abs/2009.03072, 2020.

[16] S. Conti, A. Garroni, and M. Ortiz, The line-tension approximation as the dilute limit of linear-elastic dislocations, Arch. Ration. Mech. Anal., 218 (2015), pp. 699-755. · Zbl 1457.35079

[17] L. De Luca, A. Garroni, and M. Ponsiglione, \(\Gamma \)-convergence analysis of systems of edge dislocations: The self energy regime, Arch. Ration. Mech. Anal., 206 (2012), pp. 885-910. · Zbl 1366.74006

[18] S. Fanzon, M. Palombaro, and M. Ponsiglione, A variational model for dislocations at semi-coherent interfaces, J. Nonlinear Sci., 27 (2017), pp. 1435-1461. · Zbl 1386.74026

[19] S. Fanzon, M. Palombaro, and M. Ponsiglione, Derivation of linearized polycrystals from a two-dimensional system of edge dislocations, SIAM J. Math. Anal., 51 (2019), pp. 3956-3981, https://doi.org/10.1137/18M118726X. · Zbl 1425.74071

[20] S. Fanzon, M. Ponsiglione, and R. Scala, Uniform distribution of dislocations in Peierls-Nabarro models for semi-coherent interfaces, Calc. Var. Partial Differential Equations, 59 (2020), 141. · Zbl 1440.74263

[21] G. Friesecke, R. D. James, and S. Müller, A theorem on geometric rigidity and the derivation of nonlinear plate theory from three-dimensional elasticity, Comm. Pure Appl. Math., 55 (2002), pp. 1461-1506. · Zbl 1021.74024

[22] G. Friesecke, R. D. James, and S. Müller, A hierarchy of plate models derived from nonlinear elasticity by gamma-convergence, Arch. Ration. Mech. Anal., 180 (2006), pp. 183-236. · Zbl 1100.74039

[23] A. Garroni, G. Leoni, and M. Ponsiglione, Gradient theory for plasticity via homogenization of discrete dislocations, J. Eur. Math. Soc. (JEMS), 12 (2010), pp. 1231-1266. · Zbl 1200.74017

[24] A. Garroni and S. Müller, A variational model for dislocations in the line tension limit, Arch. Ration. Mech. Anal., 181 (2006), pp. 535-578. · Zbl 1158.74365

[25] J. Ginster, Plasticity as the \(\Gamma \)-limit of a two-dimensional dislocation energy: The critical regime without the assumption of well-separateness, Arch. Ration. Mech. Anal., 233 (2019), pp. 1253-1288. · Zbl 1460.74009

[26] J. Hirth and J. Lothe, Theory of Dislocations, 2nd ed., Wiley, New York, 1982. · Zbl 1365.82001

[27] T. Hudson, An Existence Result for Discrete Dislocation Dynamics in Three Dimensions, preprint, https://arxiv.org/abs/1806.00304, 2018.

[28] T. Hudson and C. Ortner, Existence and stability of a screw dislocation under anti-plane deformation, Arch. Ration. Mech. Anal, 213 (2014), pp. 887-929. · Zbl 1342.82146

[29] T. Hudson and C. Ortner, Analysis of stable screw dislocation configurations in an antiplane lattice model, SIAM J. Math. Anal., 47 (2015), pp. 291-320, https://doi.org/10.1137/140964436. · Zbl 1317.74035

[30] R. L. Jerrard and H. M. Soner, The Jacobian and the Ginzburg-Landau energy, Calc. Var. Partial Differential Equations, 14 (2002), pp. 151-191. · Zbl 1034.35025

[31] M. Koslowsky, A. M., Cuitin⊠o, and M. Ortiz, A phase-field theory of dislocation dynamics, strain hardening and hysteresis in ductile single crystal, J. Mech. Phys. Solids, 50 (2002), pp. 2597-2635. · Zbl 1094.74563

[32] M. Koslowsky and M. Ortiz, A multi-phase field model of planar dislocation networks, Model. Simul. Mat. Sci. Eng., 12 (2004), pp. 1087-1097.

[33] G. Lauteri and S. Luckhaus, An Energy Estimate for Dislocation Configurations and the Emergence of Cosserat-Type Structures in Metal Plasticity, preprint, https//arxiv.org/abs/1608.06155, 2017.

[34] G. Lauteri and S. Luckhaus, Geometric Rigidity Estimates for Incompatible Fields in Dimension \(\ge3\), preprint, https://arxiv.org/abs/1703.0328 2017.

[35] I. Lucardesi, M. Morandotti, R. Scala, and D. Zucco, Confinement of dislocations inside a crystal with a prescribed external strain, Riv. Mat. Univ. Parma, 9 (2018), pp. 283-327. · Zbl 1417.74007

[36] M. G. Mora, M. A. Peletier, and L. Scardia, Convergence of interaction-driven evolutions of dislocations with Wasserstein dissipation and slip-plane confinement, SIAM J. Math. Anal., 49 (2017), pp. 4149-4205, https://doi.org/10.1137/16M1096098. · Zbl 1378.49012

[37] M. G. Mora, L. Rondi, and L. Scardia, The equilibrium measure for a nonlocal dislocation energy, Comm. Pure Appl. Math., 72 (2019), pp. 136-158. · Zbl 1412.49004

[38] S. Müller and M. Palombaro, Existence of minimizers for a polyconvex energy in a crystal with dislocations, Calc. Var. Partial Differential Equations, 31 (2008), pp. 473-482. · Zbl 1134.74015

[39] S. Müller and M. Palombaro, Derivation of a rod theory for biphase materials with dislocations at the interface, Calc. Var. Partial Differential Equations, 48 (2013), pp. 315-335. · Zbl 1274.74064

[40] S. Müller, L. Scardia, and C. I. Zeppieri, Geometric rigidity for incompatible fields and an application to strain-gradient

plasticity, Indiana Univ. Math. J., 63 (2014), pp. 1365-1396. · Zbl 1309.49012

[41] M. Ponsiglione, Elastic energy stored in a crystal induced by screw dislocations: From discrete to continuous, SIAM J. Math. Anal., 39 (2007), pp. 449-469, https://doi.org/10.1137/060657054. · Zbl 1135.74037

[42] E. Sandier and S. Serfaty, Vortices in the Magnetic Ginzburg-Landau Model, Birkhäuser Boston, Boston, MA, 2007. · Zbl 1112.35002

[43] E. Sandier and S. Serfaty, From the Ginzburg-Landau model to vortex lattice problems, Comm. Math. Phys., 313 (2012), pp. 635-743. · Zbl 1252.35034

[44] R. Scala and N. Van Goethem, A variational approach to single crystals with dislocations, SIAM J. Math. Anal., 51 (2019), pp. 489-531, https://doi.org/10.1137/18M1166572. · Zbl 1408.49012

[45] R. Scala and N. Van Goethem, Analytic and geometric properties of dislocation singularities, Proc. Roy. Soc. Edinburgh Sect. A, 150 (2020), pp. 1609-1651, https://doi.org/10.1017/prm.2018.57. · Zbl 1443.58003

[46] L. Scardia and C. I. Zeppieri, Line-tension model for plasticity as the \(\Gamma \)-limit of a nonlinear dislocation energy, SIAM J. Math. Anal., 44 (2012), pp. 2372-2400, https://doi.org/10.1137/110824851. · Zbl 1264.49012