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**Invariant Gibbs dynamics for the dynamical sine-Gordon model.** (English) Zbl 1473.35363  
Proc. R. Soc. Edinb., Sect. A, Math. 151, No. 5, 1450-1466 (2021).

Summary: In this note, we study the hyperbolic stochastic damped sine-Gordon equation (SdSG), with a parameter  $\beta^2 > 0$ , and its associated Gibbs dynamics on the two-dimensional torus. After introducing a suitable renormalization, we first construct the Gibbs measure in the range  $0 < \beta^2 < 4\pi$  via the variational approach due to *N. Barashkov* and *M. Gubinelli* [Duke Math. J. 169, No. 17, 3339–3415 (2020; Zbl 07292332)]. We then prove almost sure global well-posedness and invariance of the Gibbs measure under the hyperbolic SdSG dynamics in the range  $0 < \beta^2 < 2\pi$ . Our construction of the Gibbs measure also yields almost sure global well-posedness and invariance of the Gibbs measure for the parabolic sine-Gordon model in the range  $0 < \beta^2 < 4\pi$ .

**MSC:**

- [35L71](#) Second-order semilinear hyperbolic equations
- [35L20](#) Initial-boundary value problems for second-order hyperbolic equations
- [35R60](#) PDEs with randomness, stochastic partial differential equations
- [60H15](#) Stochastic partial differential equations (aspects of stochastic analysis)

Cited in 1 Document

**Keywords:**

stochastic sine-Gordon equation; dynamical sine-Gordon model; renormalization; white noise; Gibbs measure; Gaussian multiplicative chaos; periodic boundary conditions

**Full Text:** [DOI](#) [arXiv](#)

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