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**Bounds for Sobolev embedding constants in non-simply connected planar domains.** (English)

Zbl 1473.35012
Ferone, Vincenzo (ed.) et al., Geometric properties for parabolic and elliptic PDE's. Contributions of the 6th Italian-Japanese workshop, Cortona, Italy, May 20–24, 2019. Cham: Springer. Springer INdAM Ser. 47, 103-125 (2021).

Summary: In a bounded non-simply connected planar domain $\Omega$, with a boundary split in an interior part and an exterior part, we obtain bounds for the embedding constants of some subspaces of $H^1(\Omega)$ into $L^p(\Omega)$ for any $p > 1$, $p \neq 2$. The subspaces contain functions which vanish on the interior boundary and are constant (possibly zero) on the exterior boundary. We also evaluate the precision of the obtained bounds in the limit situation where the interior part tends to disappear and we show that it does not depend on $p$. Moreover, we emphasize the failure of symmetrization techniques in these functional spaces. In simple situations, a new phenomenon appears: the existence of a break even surface separating masses for which symmetrization increases/decreases the Dirichlet norm. The question whether a similar phenomenon occurs in more general situations is left open.

For the entire collection see [Zbl 1471.35003].

**MSC:**

35A23 Inequalities applied to PDEs involving derivatives, differential and integral operators, or integrals
35J05 Laplace operator, Helmholtz equation (reduced wave equation), Poisson equation
35J25 Boundary value problems for second-order elliptic equations
46E35 Sobolev spaces and other spaces of "smooth" functions, embedding theorems, trace theorems



**Keywords:**

pyramidal functions; symmetrization; bounded non-simply connected planar domain

**Full Text:** DOI

**References:**

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