

Bandyopadhyay, Ritwik; Prithvirajan, Veerappan; Peralta, Alonso D.; Sangid, Michael D. Microstructure-sensitive critical plastic strain energy density criterion for fatigue life prediction across various loading regimes. (English) [Zbl 1472.74028](#)
Proc. R. Soc. Lond., A, Math. Phys. Eng. Sci. 476, No. 2236, Article ID 20190766, 23 p. (2020).

Summary: In the present work, we postulate that a critical value of the stored plastic strain energy density (SPSED) is associated with fatigue failure in metals and is independent of the applied load. Unlike the classical approach of estimating the (homogenized) SPSED as the cumulative area enclosed within the macroscopic stress-strain hysteresis loops, we use crystal plasticity finite element simulations to compute the (local) SPSED at each material point within polycrystalline aggregates of a nickel-based superalloy. A Bayesian inference method is used to calibrate the critical SPSED, which is subsequently used to predict fatigue lives at nine different strain ranges, including strain ratios of 0.05 and -1 , using nine statistically equivalent microstructures. For each strain range, the predicted lives from all simulated microstructures follow a lognormal distribution. Moreover, for a given strain ratio, the predicted scatter is seen to be increasing with decreasing strain amplitude; this is indicative of the scatter observed in the fatigue experiments. Finally, the lognormal mean lives at each strain range are in good agreement with the experimental evidence. Since the critical SPSED captures the experimental data with reasonable accuracy across various loading regimes, it is hypothesized to be a material property and sufficient to predict the fatigue life.

MSC:

74C05 Small-strain, rate-independent theories of plasticity (including rigid-plastic and elasto-plastic materials)

Keywords:

Bayesian inference method; Markov chain Monte Carlo; metropolis-Hastings algorithm; fatigue life; crystal plasticity finite element method; plastic strain energy density; mechanical engineering; mechanics; materials science

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