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**Analysis of dual Bernstein operators in the solution of the fractional convection-diffusion equation arising in underground water pollution.** (English) Zbl 1472.35441

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Summary: The Bernstein operators (BO) are not orthogonal, but they have duals, which are obtained by a linear combination of BO. In recent years dual BO have been adopted in computer graphics, computer aided geometric design, and numerical analysis. This paper presents a numerical method based on the Bernstein operational matrices to solve the time-space fractional convection-diffusion equation. A generalization of the derivative matrix operator of fractional order and the error analysis are discussed. Numerical examples compare the proposed approach with previous works, showing that the method is more accurate and efficient.

**MSC:**

[35R11](#) Fractional partial differential equations

[35K20](#) Initial-boundary value problems for second-order parabolic equations

[92D40](#) Ecology

**Keywords:**

dual Bernstein operators; fractional convection-diffusion equation; error analysis; operational matrix

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