Cortázar, Carmen; Quirós, Fernando; Wolanski, Noemí

A heat equation with memory: large-time behavior. (English) Zbl 1472.35049

J. Funct. Anal. 281, No. 9, Article ID 109174, 40 p. (2021).

Summary: We study the large-time behavior in all L^p norms of solutions to a heat equation with a Caputo α-time derivative posed in $\mathbb{R}^N$ $(0 < \alpha < 1)$. These are known as subdiffusion equations. The initial data are assumed to be integrable, and, when required, to be also in L^p.

We find that the decay rate in all L^p norms, $1 \le p \le \infty$, depends greatly on the space-time scale under consideration. This result explains in particular the so called "critical dimension phenomenon" (cf. [*J. Kemppainen* et al., Math. Ann. 366, No. 3–4, 941–979 (2016; Zbl 1354.35178)]).

Moreover, we find the final profiles (that strongly depend on the scale). The most striking result states that in compact sets the final profile (in all L^p norms) is a multiple of the Newtonian potential of the initial datum.

Our results are very different from the ones for classical diffusion equations and show that, in accordance with the physics they have been proposed for, these are good models for particle systems with sticking and trapping phenomena or fluids with memory.

MSC:

35B40 Asymptotic behavior of solutions to PDEs
35A08 Fundamental solutions to PDEs
35K15 Initial value problems for second-order parabolic equations
35R11 Fractional partial differential equations

Keywords:

heat equation with nonlocal time derivative; Caputo derivative; asymptotic behavior

Full Text: DOI arXiv

References:

[1] Allen, M.; Caffarelli, L.; Vasseur, A., A parabolic problem with a fractional time derivative, Arch. Ration. Mech. Anal., 221, 2, 603-630 (2016) · Zbl 1338.35428

[2] Benilan, Ph.; Brezis, H.; Crandall, M. G., A semilinear equation in \(L^1(\mathbb{R}^N)\), Ann. Sc. Norm. Super. Pisa, Cl. Sci. (4), 2, 4, 523-555 (1975) · Zbl 0314.35077

[3] Caputo, M., Linear models of dissipation whose Q is almost frequency independent-II, Geophys. J. R. Astron. Soc., 13, 529-539 (1967)

[4] Caputo, M., Diffusion of fluids in porous media with memory, Geothermics, 28, 113-130 (1999)

[5] Cortázar, C.; Elgueta, M.; Quirós, F.; Wolanski, N., Asymptotic behavior for a nonlocal diffusion equation in exterior domains: the critical two-dimensional case, J. Math. Anal. Appl., 436, 1, 586-610 (2016) · Zbl 1332.35158

[6] Cortázar, C.; Elgueta, M.; Quirós, F.; Wolanski, N., Asymptotic behavior for a one-dimensional nonlocal diffusion equation in exterior domains, SIAM J. Math. Anal., 48, 3, 1549-1574 (2016) · Zbl 1381.35203

[7] Cortázar, C.; Quirós, F.; Wolanski, N., Near-field asymptotics for the porous medium equation in exterior domains. The critical two-dimensional case, SIAM J. Math. Anal., 50, 3, 2664-2680 (2018) · Zbl 1393.35114

[8] Cortázar, C.; Quirós, F.; Wolanski, N., Large-time behavior for a fully nonlocal heat equation, Vietnam J. Math. (2021), Available at

[9] Cortázar, C.; Quirós, F.; Wolanski, N., Decay/growth rates for inhomogeneous heat equations with memory. The case of large dimensions, preprint. Available at

[10] C. Cortázar, F. Quirós, N. Wolanski, Decay/growth rates for inhomogeneous heat equations with memory. The case of small dimensions, preprint. · Zbl 1393.35114

[11] C. Cortázar, F. Quirós, N. Wolanski, Asymptotic profiles for inhomogeneous heat equations with memory, preprint. · Zbl 1360.35023

[12] Dier, D.; Kemppainen, J.; Siljander, J.; Zacher, R., On the parabolic Harnack inequality for non-local diffusion equations, Math. Z., 295, 3-4, 1751-1769 (2020) · Zbl 1446.35246

[13] Dipierro, S.; Valdinoci, E.; Vespri, V., Decay estimates for evolutionary equations with fractional time-diffusion, J. Evol. Equ., 19, 2, 435-462 (2019) · Zbl 1461.35049

[14] Duoandikoetxea, J.; Zuazua, E., Moments, masses de Dirac et décomposition de fonctions, C. R. Acad. Sci. Paris, Sér. I Math., 315, 6, 693-698 (1992), (in French, English, French summary) [Moments, Dirac deltas and expansion of functions] · Zbl 0755.45019

[15] Dzherbashyan, M. M.; Nersesian, A. B., Fractional derivatives and the Cauchy problem for differential equations of fractional order, Izv. Nats. Akad. Nauk Armenii Mat., 3, 3-29 (1968), (in Russian)

[16] Eidelman, S. D.; Kochubei, A. N., Cauchy problem for fractional diffusion equations, J. Differ. Equ., 199, 2, 211-255 (2004) · Zbl 1068.35037

[17] Gerasimov, A. N., A generalization of linear laws of deformation and its application to problems of internal friction, Akad. Nauk SSSR, Prikl. Mat. Meh., 12, 251-260 (1948), (in Russian)

[18] Gripenberg, G., Volterra integro-differential equations with accretive nonlinearity, J. Differ. Equ., 60, 1, 57-79 (1985) · Zbl 0575.45013

[19] Gross, B., On creep and relaxation, J. Appl. Phys., 18, 212-221 (1947)

[20] Herraiz, L., Asymptotic behaviour of solutions of some semilinear parabolic problems, Ann. Inst. Henri Poincaré, Anal. Non Linéaire, 16, 1, 49-105 (1999) · Zbl 0918.35025

[21] Kemppainen, J.; Siljander, J.; Vergara, V.; Zacher, R., Decay estimates for time-fractional and other non-local in time subdiffusion equations in \(\mathbb{R}^d\), Math. Ann., 366, 3-4, 941-979 (2016) · Zbl 1354.35178

[22] Kemppainen, J.; Siljander, J.; Zacher, R., Representation of solutions and large-time behavior for fully nonlocal diffusion equations, J. Differ. Equ., 263, 1, 149-201 (2017) · Zbl 1366.35218

[23] Kochubeĭ, A. N., Diffusion of fractional order, Differ. Uravn.. Differ. Uravn., Differ. Equ., 26, 4, 485-492 (1990), translation in · Zbl 0729.35064

[24] Liouville, J., Mémoire sur quelques questions de géometrie et de mécanique, et sur un nouveau genre de calcul pour résoudre ces questions, J. Éc. Polytech., 13, 1-69 (1832)

[25] Meerschaert, M. M.; Nane, E.; Vellaisamy, P., Fractional Cauchy problems on bounded domains, Ann. Probab., 37, 3, 979-1007 (2009) · Zbl 1247.60078

[26] Meerschaert, M. M.; Sikorskii, A., Stochastic Models for Fractional Calculus, De Gruyter Studies in Mathematics, vol. 43 (2019), De Gruyter: De Gruyter Berlin, 978-3-11-056024-978-3-11-4, 978-3-11-055914-978-3-11-9 · Zbl 07002290

[27] Metzler, R.; Klafter, J., The random walk's guide to anomalous diffusion: a fractional dynamics approach, Phys. Rep., 339, 1 (2000), 77 pp. · Zbl 0984.82032

[28] Nakagawa, J.; Sakamoto, K.; Yamamoto, M., Overview to mathematical analysis for fractional diffusion equations—new mathematical aspects motivated by industrial collaboration, J. Math-for-Ind., 2A, 99-108 (2010) · Zbl 1206.35247

[29] Prüss, J., Evolutionary Integral Equations and Applications (1993), Modern Birkhäuser Classics. Birkhäuser/Springer: Modern Birkhäuser Classics. Birkhäuser/Springer Basel AG, Basel, [2012] reprint of the 1993 edition · Zbl 0793.45014

[30] Rabotnov, Yu. N., Polzuchest Elementov Konstruktsii, Creep Problems in Structural Members (1969), Nauka: Nauka Moscow: North-Holland: Nauka: Nauka Moscow: North-Holland Amsterdam, (in Russian); English translation:

[31] Shlesinger, M. F.; Klafter, J.; Wong, Y. M., Random walks with infinite spatial and temporal moments, J. Stat. Phys., 27, 3, 499-512 (1982) · Zbl 0521.60080

[32] Vergara, V.; Zacher, R., Optimal decay estimates for time-fractional and other nonlocal subdiffusion equations via energy methods, SIAM J. Math. Anal., 47, 1, 210-239 (2015) · Zbl 1317.45006

[33] Zygmund, A., On a theorem of Marcinkiewicz concerning interpolation of operations, J. Math. Pures Appl. (9), 35, 223-248 (1956) · Zbl 0070.33701