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Asymptotic expansions with exponential, power, and logarithmic functions for non-autonomous nonlinear differential equations. (English) Zbl 1472.34021

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Summary: This paper develops further and systematically the asymptotic expansion theory that was initiated by *C. Foias* and *J. C. Saut* [Ann. Inst. Henri Poincaré, Anal. Non Linéaire 4, 1–47 (1987; Zbl 0635.35075)]. We study the long-time dynamics of a large class of dissipative systems of nonlinear ordinary differential equations with time-decaying forcing functions. The nonlinear term can be, but not restricted to, any smooth vector field which, together with its first derivative, vanishes at the origin. The forcing function can be approximated, as time tends to infinity, by a series of functions which are coherent combinations of exponential, power and iterated logarithmic functions. We prove that any decaying solution admits an asymptotic expansion, as time tends to infinity, corresponding to the asymptotic structure of the forcing function. Moreover, these expansions can be generated by more than two base functions and go beyond the polynomial formulation imposed in previous work.

MSC:

34A25 Analytical theory of ordinary differential equations: series, transformations, transforms, operational calculus, etc.
34D05 Asymptotic properties of solutions to ordinary differential equations
37C60 Nonautonomous smooth dynamical systems
41A60 Asymptotic approximations, asymptotic expansions (steepest descent, etc.)

Keywords:

asymptotic expansions; long-time dynamics; non-autonomous systems; dissipative dynamical systems; perturbations

Full Text: DOI arXiv

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