Mirhosseinkhani, Ghasem; Nazari, Narges
The category of topological De Morgan molecular lattices. (English) Zbl 1472.06013
J. Math. Ext. 14, No. 3, 133-149 (2020).

There exist numerous approaches to point-free topology. For example, [*G. Wang*, Fuzzy Sets Syst. 47, No. 3, 351–376 (1992; Zbl 0783.54032)] considered the theory of topological molecular lattices, where a molecular lattice stands for a complete completely distributive lattice. A point-free analogue of a topological space is then a molecular lattice L, equipped with a subset $\tau \subseteq L$ (called *co-topology* on L), which is closed under finite joins and arbitrary meets. As an analogue of a continuous map, one considers *continuous generalized order homomorphisms* $f : (L_1, \tau_1) \to (L_2, \tau_2)$, namely, join-preserving maps $f : L_1 \to L_2$, which possess a join-preserving right adjoint map $\hat{f} : L_2 \to L_1$ such that $\hat{f}(a) \in \tau_1$ for every $a \in \tau_2$ (*continuity*).

The present authors continue the study of *N. Nazari* and *G. Mirhosseinkhani* [Sahand Commun. Math. Anal. 10, No. 1, 1–15 (2018; Zbl 1413.06009)], where they introduced generalized topological molecular lattices, considering subsets τ of molecular lattices closed under finite meets and arbitrary joins (namely, dualizing the setting of Wang [loc. cit.]), which were called *topologies* on L. Moreover, in the paper under review, they take molecular lattices L together with a *pseudocomplementation operation* $(-)^*$ defined by $a^* = \bigvee\{b \in L \mid a \wedge b = \perp_L\}$ [*T. S. Blyth*, Lattices and ordered algebraic structures. London: Springer (2005; Zbl 1073.06001)], and, additionally, suppose that such molecular lattices L satisfy the first De Morgan law, i.e., $(\bigvee S)^* = \bigwedge_{s\in S} s^*$ for every subset $S \subseteq L$. The authors define a particular category of topological De Morgan molecular lattices (**TDML**), and show that the category of topological spaces and continuous maps is isomorphic to both a reflective and a coreflective subcategory of **TDML**. In the second part of the paper, the authors provide an explicit description of (co)equalizers and (co)products in the category **TDML**.

The paper is well written (but somewhat technical), provides the most essential parts of its required preliminaries, and will be of interest to all those researchers who study categorical approaches to point-free topology.

Reviewer: Sergejs Solovjovs (Praha)

MSC:

06D30 De Morgan algebras, Łukasiewicz algebras (lattice-theoretic aspects)
06A15 Galois correspondences, closure operators (in relation to ordered sets)
06D10 Complete distributivity
18B35 Preorders, orders, domains and lattices (viewed as categories)
18F70 Frames and locales, pointfree topology, Stone duality

Keywords:

adjoint functor; Boolean algebra; (co)equalizers; completely distributive lattice; coprime element; (co)products; (co)reflective subcategory; De Morgan lattice; forgetful functor; molecular lattice; Stone algebra

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