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A semi-analytical solution for multilayer diffusion in a composite medium consisting of a large number of layers. (English) Zbl 1471.74014

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Summary: Diffusion in a composite slab consisting of a large number of layers provides an ideal prototype problem for developing and analysing two-scale modelling approaches for heterogeneous media. Numerous analytical techniques have been proposed for solving the transient diffusion equation in a one-dimensional composite slab consisting of an arbitrary number of layers. Most of these approaches, however, require the solution of a complex transcendental equation arising from a matrix determinant for the eigenvalues that is difficult to solve numerically for a large number of layers. To overcome this issue, in this paper, we present a semi-analytical method based on the Laplace transform and an orthogonal eigenfunction expansion. The proposed approach uses eigenvalues local to each layer that can be obtained either explicitly, or by solving simple transcendental equations. The semi-analytical solution is applicable to both perfect and imperfect contact at the interfaces between adjacent layers and either Dirichlet, Neumann or Robin boundary conditions at the ends of the slab. The solution approach is verified for several test cases and is shown to work well for a large number of layers. The work is concluded with an application to macroscopic modelling where the solution of a fine-scale multilayered medium consisting of two hundred layers is compared against an "up-scale" variant of the same problem involving only ten layers.

MSC:

74E30 Composite and mixture properties
74H05 Explicit solutions of dynamical problems in solid mechanics
74H10 Analytic approximation of solutions (perturbation methods, asymptotic methods, series, etc.) of dynamical problems in solid mechanics
74H15 Numerical approximation of solutions of dynamical problems in solid mechanics

Keywords:

diffusion; multilayer; composite medium; macroscale model; analytical solution; finite volume method

Full Text: DOI

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