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**Codenseness and openness with respect to an interior operator.** (English) Zbl 1471.18003  
Appl. Categ. Struct. 29, No. 2, 235-248 (2021).

This paper provides an interesting generalization of the notions of codenseness and openness with respect to an interior operator on a finitely complete category.

First of all, the authors introduce some basic properties of the notion of codenseness with respect to an interior operator  $i$  on a finitely complete category  $\mathbb{C}$  (with a proper  $(\mathcal{E}, \mathcal{M})$ -factorization system for morphism) with respect to  $\mathcal{M}$ .

Secondly, they define  $i$ -codense morphisms in this category and provide some important stability properties of the class of  $i$ -codense morphisms in  $\mathbb{C}$ .

Thirdly, using the notion of an open morphism with respect to an interior operator introduced by *G. Castellini* [Categorical closure operators. Boston, MA: Birkhäuser (2003; Zbl 1045.18001)], the authors give a number of new characterizations and some properties of this class of morphisms in  $\mathbb{C}$ .

Finally, the authors present a notion of quasi-open morphisms with respect to an interior operator  $i$ , and show that the quasi  $i$ -open morphisms of  $\mathbb{C}$  are characterized as the morphisms which reflect  $i$ -codensity.

Reviewer: Joaquín Luna-Torres (Cartagena)

**MSC:**

- 18A20 Epimorphisms, monomorphisms, special classes of morphisms, null morphisms
- 06A15 Galois correspondences, closure operators (in relation to ordered sets)
- 54B30 Categorical methods in general topology

**Keywords:**

interior operator; codenseness; openness; quasi-openness

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**References:**

- [1] Adámek, J.; Herrlich, H.; Strecker, G., Abstract and Concrete Categories. Pure and Applied Mathematics (1990), New York: Wiley, New York · Zbl 0695.18001
- [2] Assfaw, F.: Interior Operators and Their Applications. Ph.D. thesis, University of the Western Cape (2019)
- [3] Assfaw, F., Holgate, D.: Hereditary Interior Operators. Submitted for publication (2019)
- [4] Castellini, G., Categorical Closure Operators. Mathematics: Theory & Applications (2003), Boston: Birkhäuser Boston, Inc., Boston · Zbl 1045.18001 · doi:10.1007/978-0-8176-8234-7
- [5] Castellini, G., Interior operators in a category: idempotency and heredity, Topol. Appl., 158, 17, 2332-2339 (2011) · Zbl 1230.18002 · doi:10.1016/j.topol.2011.06.030
- [6] Castellini, G., Interior operators, open morphisms and the preservation property, Appl. Categ. Struct., 23, 3, 311-322 (2015) · Zbl 1316.18002 · doi:10.1007/s10485-013-9337-4
- [7] Castellini, G., Some remarks on interior operators and the functional property, Quaest. Math., 39, 2, 275-287 (2016) · Zbl 1420.18001 · doi:10.2989/16073606.2015.1070379
- [8] Castellini, G.; Murcia, E., Interior operators and topological separation, Topol. Appl., 160, 12, 1476-1485 (2013) · Zbl 1284.54024 · doi:10.1016/j.topol.2013.05.023
- [9] Castellini, G.; Ramos, J., Interior operators and topological connectedness, Quaest. Math., 33, 3, 290-304 (2010) · Zbl 1274.54067 · doi:10.2989/16073606.2010.507322
- [10] Clementino, M., Giuli, E., Tholen, W.: A functional approach to general topology. In: Categorical Foundations, Volume 97 of Encyclopedia of Mathematics and Applications., pp. 103-163. Cambridge University Press, Cambridge (2004) · Zbl 1059.54012
- [11] Dikranjan, D., Giuli, E.: Closure operators. I. In: Proceedings of the 8th International Conference on Categorical Topology (L'Aquila, 1986), vol. 27, pp. 129-143 (1987) · Zbl 0634.54008
- [12] Dikranjan, D., Giuli, E., Tholen, W.: Closure operators. II. Categorical Topology and Its Relation to Analysis. Algebra and Combinatorics (Prague, 1988), pp. 297-335. World Scientific Publishing, Teaneck (1989)

- [13] Dikranjan, D., Tholen, W.: *Categorical Structure of Closure Operators*. Mathematics and its Applications, vol. 346. Kluwer Academic Publishers Group, Dordrecht (1995) · [Zbl 0853.18002](#)
- [14] Dikranjan, D.; Tholen, W., Dual closure operators and their applications, *J. Algebra*, 439, 373-416 (2015) · [Zbl 1320.18002](#) · [doi:10.1016/j.jalgebra.2015.04.041](#)
- [15] Engelking, R., *General Topology*, Volume 6 of Sigma Series in Pure Mathematics (1989), Berlin: Heldermann Verlag, Berlin
- [16] Herrlich, H.; Strecker, GE,  $\mathcal{H}$ -closed spaces and reflective subcategories, *Math. Ann.*, 177, 302-309 (1968) · [Zbl 0157.29104](#) · [doi:10.1007/BF01350722](#)
- [17] Holgate, D.; Šlapal, J., Categorical neighborhood operators, *Topol. Appl.*, 158, 17, 2356-2365 (2011) · [Zbl 1232.54018](#) · [doi:10.1016/j.topol.2011.06.031](#)
- [18] Kao, KS, A note on  $(M_1)$ -spaces, *Pac. J. Math.*, 108, 1, 121-128 (1983) · [Zbl 0487.54029](#) · [doi:10.2140/pjm.1983.108.121](#)
- [19] Kim, JW, A note on quasi-open maps, *J. Korea Soc. Math. Educ. Ser. B Pure Appl. Math.*, 5, 1, 1-3 (1998)
- [20] Luna-Torres, J.; Ochoa, COC, Interior operators and topological categories, *Adv. Appl. Math. Sci.*, 10, 2, 189-206 (2011) · [Zbl 1245.18001](#)
- [21] Mardešić, S., Papić, P.: Continuous images of ordered compacta, the Suslin property and diadic compacta. *Glasnik Mat.-Fiz. Astronom. Društvo Mat. Fiz. Hrvatske SER. II* 17, 3-25 (1963). 1962 · [Zbl 0119.17906](#)
- [22] Razafindrakoto, A.; Holgate, D., Interior and neighbourhood, *Topol. Appl.*, 168, 144-152 (2014) · [Zbl 1321.18005](#) · [doi:10.1016/j.topol.2014.02.019](#)
- [23] Salbany, S.: *Reflective Subcategories and Closure Operators*, pp. 548-565. *Lecture Notes in Mathematics*, vol. 540 (1976) · [Zbl 0335.54003](#)
- [24] Vorster, S., Interior operators in general categories, *Quaest. Math.*, 23, 4, 405-416 (2000) · [Zbl 0974.18003](#) · [doi:10.2989/16073600009485987](#)

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