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Hopf bifurcations and transitions of two-dimensional quasi-geostrophic flows. (English)

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Summary: This study examines the Hopf (double Hopf) bifurcations and transitions of two dimensional quasi-geostrophic (QG) flows that model various large-scale oceanic and atmospheric circulations. Using the Kolmogorov function to represent an external forcing in the tropical region, it is shown that the equilibrium of the QG model loses its stability if the combination of the Rossby number, the Ekman number, and the eddy viscosity satisfies a specific condition. Further use of the center manifold technique reveals two different types of the dynamical transition from either a pair of simple complex eigenvalues or a double pair of complex conjugate eigenvalues. These dynamical transitions are confirmed in the numerical analyses of the QG dynamics at the equilibrium, which capture Hopf (double Hopf) bifurcations due to the existence of a nonzero imaginary part of the first eigenvalue. The transition from a pair of simple complex eigenvalues is of particular interest, because it indicates the existence of a stable periodic pattern that is similar to atmospheric easterly waves and related tropical cyclone formation in the tropical atmosphere.

MSC:

37N10 Dynamical systems in fluid mechanics, oceanography and meteorology
37L10 Normal forms, center manifold theory, bifurcation theory for infinite-dimensional dissipative dynamical systems
37L15 Stability problems for infinite-dimensional dissipative dynamical systems
76E20 Stability and instability of geophysical and astrophysical flows
76U60 Geophysical flows

Keywords:

dynamical transition; breakdown; instability; Hopf bifurcation; barotropic dynamics

Full Text: DOI

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