This memoir deals with generalisations of the classical Eichler-Shimura isomorphism. Recall that given a cofinite Fuchsian group $\Gamma$ (for example $\Gamma = \text{SL}_2(\mathbb{Z})$) Eichler associated, to a cuspidal holomorphic automorphic form $F$ of weight $k$ (an even positive integer), a cohomology class on $\Gamma$ with coefficients in the space of homogeneous polynomials in two variables of degree $(k-2)$. More recently cohomology classes associated with real weights have been considered, where infinite-dimensional coefficients are needed. In the present work the authors deal with complex weight and arbitrary forms (no growth condition at the cusps).

In the classical Eichler-Shimura isomorphism, the cohomological side is that of group cohomology for the Fuchsian group with values in spaces of homogeneous polynomials in 2 variables (which are the spaces for the representations of $\text{SL}_2(\mathbb{R})$, and also functions on the upper half-plane). In this book, the authors interpret automorphic forms with complex weight as cohomology classes for the Fuchsian group with coefficients in spaces of holomorphic functions on the lower half-plane (which appears to be more convenient than the upper for technical reasons). For real weights they additionally characterise cusp forms in the image. They also discuss cohomologies with further coefficients systems.

Reviewer: Jean Raimbault (Marseille)

MSC:

11F67 Special values of automorphic $L$-series, periods of automorphic forms, cohomology, modular symbols
11F75 Cohomology of arithmetic groups
11F12 Automorphic forms, one variable
22E40 Discrete subgroups of Lie groups

Keywords:
holomorphic automorphic form; Eichler integral; cohomology; mixed parabolic cohomology; period function; harmonic lift; harmonic functions; boundary germ

Full Text: DOI arXiv Link

References:


[22] Y. Choie: Rational period functions for the modular group and real quadratic fields; Illinois J. Mathematics 33.3 (1989) 495-529 · Zbl 0659.10022


[27] Y. Choie: Hecke operators on rational period functions on the Hecke groups; Results in Mathematics 25 (1994) 40-49 · Zbl 0802.11019


[31] Y. Choie, S. Lim: Eichler integrals, period relations and Jacobi forms; Math. Z. 271.3-4 (2012) 639-661 · Zbl 1251.11032


[33] H. Cohn: Variational property of cusp forms; Trans. AMS 82.1 (1956) 117-127 · Zbl 0074.30401

[34] H. Cohn, M. Knopp: Note on automorphic forms with real period polynomials; Duke Math. J. 32.1 (1965) 115-120 · Zbl 0138.31801

[35] A. Deitmar, J. Hilgert: Cohomology of arithmetic groups with infinite dimensional coefficient spaces; Documenta Math. 10 (2005) 199-216 · Zbl 1084.11024


[38] A. Deitmar: Lewis-Zagier correspondence for higher-order forms; Pac. J. Math. 249.1 (2011) 11-21 · Zbl 1314.11023


[40] N. Diamantis: Special values of higher derivatives of \( (L) \)-functions; Forum Mathematica 11.2 (1999) 229-252 · Zbl 0932.11032


I.Kra: On cohomology of kleinian groups; Acta Arithm. 11.2 (1965) 169-180 · Zbl 0148.32503


J.D.Fay: Fourier coefficient of the resolvent for a Fuchsian group; J. reine angew. Math. 293/294 (1977) 143-203 · Zbl 0352.30002


S.Fukuhara: The space of period polynomials; Acta Arithm. 82.1 (1977) 77-93


D.Goldfeld: Special values of derivatives of \(|L(s)|\)-functions; Number theory (Halifax, NS, 1994); CMS Conf. Proc. 15, Amer. Math. Soc., Providence, RI (1995) 159-171 · Zbl 0845.11021

L.Goldstein, M.Razar: The theory of Hecke integrals; Nagoya Math. J. 63 (1976), 93-121 · Zbl 0346.10011

R.C.Gunning: Factors of automorphy and other cohomology groups for Lie groups; Ann. of Math. 69.2 (1959) 314-326 · Zbl 0173.33203

R.C.Gunning: The Eichler cohomology groups and automorphic forms; Trans. AMS 100.1 (1961) 44-62 · Zbl 0142.05302


L.Hornander: An Introduction to Complex Analysis in Several Variables; van Nostrand, 1966 · Zbl 0138.06203


K.Hara, M.Kaneko, D.Zagier: Derivation and double shuffle relations for multiple zeta values; Compos. Math. 142.2 (2006) 307-338 · Zbl 1186.11053

D.Jeon, S.-Y.Kang, Chang.H.Kim: Weak Maass-Poincare series and weight 3/2 mock modular forms; J. Number Th. 133.8 · Zbl 1233.11058

K.Ihara, M.Kaneko, D.Zagier: Derivation and double shuffle relations for multiple zeta values; Compos. Math. 142.2 (2006) 307-338 · Zbl 1253.11086

Zbl 1287.11052


R.C.Gunning: Factors of automorphy and other cohomology groups for Lie groups; Ann. of Math. 69.2 (1959) 314-326 · Zbl 0173.33203


S.Lang: \(|\zeta|\); Addison-Wesley, Reading, MA-London-Amsterdam, 1975

S.Lang: Real and Functional Analysis; Springer-Verlag, 1993 · Zbl 0831.40001


W.Kohnen, D.Zagier: Moduli forms with rational periods; in Modular Forms, R.A.Rankin (ed.), Durham; Ellis Horwood, Chichester, 1984, 197-249


S.Lang: \(|\zeta|\); Addison-Wesley, Reading, MA-London-Amsterdam, 1975

S.Lang: Real and Functional Analysis; Springer-Verlag, 1993 · Zbl 0831.40001

S.Lang: \(|\zeta|\); Addison-Wesley, Reading, MA-London-Amsterdam, 1975

S.Lang: \(|\zeta|\); Addison-Wesley, Reading, MA-London-Amsterdam, 1975

S.Lang: \(|\zeta|\); Addison-Wesley, Reading, MA-London-Amsterdam, 1975

S.Lang: \(|\zeta|\); Addison-Wesley, Reading, MA-London-Amsterdam, 1975


[121] D.Zagier: Derivation and double shuffle relations for multiple zeta values; Compos. Math. 142.2 (2006) 307-338 · Zbl 1186.11053


[126] D.Zagier: Evaluation of the multiple zeta values \( \zeta(2,...,2,3,...,2) \); Ann. of Math. (2) 175 (2012) no. 2, 977-1000 · Zbl 1268.11121

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.