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**On Boolean ranges of Banaschewski functions.** (English) Zbl 1469.06010  
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**Summary:** We construct a countable lattice  $\mathcal{S}$  isomorphic to a bounded sublattice of the subspace lattice of a vector space with two non-isomorphic maximal Boolean sublattices. We represent one of them as the range of a Banaschewski function and we prove that this is not the case of the other. Hereby we solve a problem of F. Wehrung. We study coordinatizability of the lattice  $\mathcal{S}$ . We prove that although it does not contain a 3-frame, the lattice  $\mathcal{S}$  is coordinatizable. We show that the two maximal Boolean sublattices correspond to maximal abelian regular subalgebras of the coordinatizing ring.

**MSC:**

- 06B05 Structure theory of lattices
- 06A15 Galois correspondences, closure operators (in relation to ordered sets)
- 06C20 Complemented modular lattices, continuous geometries
- 06D75 Other generalizations of distributive lattices

**Keywords:**

lattice; complemented; modular; Boolean; Schmidt's construction; Banaschewski function; closure operator; adjunction

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