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Duality results and regularization schemes for Prandtl-Reuss perfect plasticity. (English)
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Summary: We consider the time-discretized problem of the quasi-static evolution problem in perfect plasticity posed in a non-reflexive Banach space. Based on a novel equivalent reformulation in a reflexive Banach space, the primal problem is characterized as a Fenchel dual problem of the classical incremental stress problem. This allows to obtain necessary and sufficient optimality conditions for the time-discrete problems of perfect plasticity. Furthermore, the consistency of a primal-dual stabilization scheme is proven. As a consequence, not only stresses, but also displacements and strains are shown to converge to a solution of the original problem in a suitable topology. The corresponding dual problem has a simpler structure and turns out to be well-suited for numerical purposes. For the resulting subproblems an efficient algorithmic approach in the infinite-dimensional setting based on the semismooth Newton method is proposed.

MSC:

74C05 Small-strain, rate-independent theories of plasticity (including rigid-plastic and elasto-plastic materials)
74S99 Numerical and other methods in solid mechanics
35Q74 PDEs in connection with mechanics of deformable solids

Keywords:

small-strain plasticity; Fenchel duality; semismooth Newton method; Tikhonov regularization; time discretization; convergence

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