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Galois connections for patterns: an algebra of labelled graphs. (English) Zbl 1467.68176

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Summary: A pattern is a generic instance of a binary constraint satisfaction problem (CSP) in which the compatibility of certain pairs of variable-value assignments may be unspecified. The notion of forbidden pattern has led to the discovery of several novel tractable classes for the CSP. However, for this field to come of age it is time for a theoretical study of the algebra of patterns. We present a Galois connection between lattices composed of sets of forbidden patterns and sets of generic instances, and investigate its consequences. We then extend patterns to augmented patterns and exhibit a similar Galois connection. Augmented patterns are a more powerful language than flat (i.e. non-augmented) patterns, as we demonstrate by showing that, for any $k \geq 1$, instances with tree-width bounded by k cannot be specified by forbidding a finite set of flat patterns but can be specified by a finite set of augmented patterns. A single finite set of augmented patterns can also describe the class of instances such that each instance has a weak near-unanimity polymorphism of arity k (thus covering all tractable language classes). We investigate the power of forbidding augmented patterns and discuss their potential for describing new tractable classes.

For the entire collection see [\[Zbl 1467.68005\]](#).

MSC:

- 68T30** Knowledge representation
- 05C78** Graph labelling (graceful graphs, bandwidth, etc.)
- 06A15** Galois correspondences, closure operators (in relation to ordered sets)
- 68R10** Graph theory (including graph drawing) in computer science

Keywords:

[constraint satisfaction](#); [tractability](#); [forbidden patterns](#); [Galois connection](#); [lattices](#)

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