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Homogenization in BV of a model for layered composites in finite crystal plasticity. (English)

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Summary: In this work, we study the effective behavior of a two-dimensional variational model within finite crystal plasticity for high-contrast bilayered composites. Precisely, we consider materials arranged into periodically alternating thin horizontal strips of an elastically rigid component and a softer one with one active slip system. The energies arising from these modeling assumptions are of integral form, featuring linear growth and non-convex differential constraints. We approach this non-standard homogenization problem via Gamma-convergence. A crucial first step in the asymptotic analysis is the characterization of rigidity properties of limits of admissible deformations in the space BV of functions of bounded variation. In particular, we prove that, under suitable assumptions, the two-dimensional body may split horizontally into finitely many pieces, each of which undergoes shear deformation and global rotation. This allows us to identify a potential candidate for the homogenized limit energy, which we show to be a lower bound on the Gamma-limit. In the framework of non-simple materials, we present a complete Gamma-convergence result, including an explicit homogenization formula, for a regularized model with an anisotropic penalization in the layer direction.

MSC:

49S05	Variational principles of physics
49J45	Methods involving semicontinuity and convergence; relaxation
74E15	Crystalline structure
74C15	Large-strain, rate-independent theories of plasticity (including nonlinear plasticity)

Keywords:

finite crystal plasticity; non-simple materials

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References:

[1] G. Alberti, Rank one property for derivatives of functions with bounded variation, Proc. Roy. Soc. Edinburgh Sect. A 123 (1993), no. 2, 239-274. · Zbl 0791.26008

[2] L. Ambrosio, N. Fusco and D. Pallara, Functions of Bounded Variation and Free Discontinuity Problems, Oxford Math. Monogr., The Clarendon Press, Oxford, 2000. · Zbl 0957.49001

[3] S. Amstutz and N. Van Goethem, Incompatibility-governed elasto-plasticity for continua with dislocations, Proc. R. Soc. Lond. A Math. Phys. Eng. Sci. 473 (2017), no. 2199, Article ID 20160734. · Zbl 1404.74022

[4] H. Attouch, Variational Convergence for Functions and Operators, Appl. Math. Ser., Pitman, Boston, 1984. · Zbl 0561.49012

[5] J. M. Ball, J. C. Currie and P. J. Olver, Null Lagrangians, weak continuity, and variational problems of arbitrary order, J. Funct. Anal. 41 (1981), no. 2, 135-174. · Zbl 0459.35020

[6] A. C. Barroso, J. Matias, M. Morandotti and D. R. Owen, Second-order structured deformations: Relaxation, integral representation and applications, Arch. Ration. Mech. Anal. 225 (2017), no. 3, 1025-1072. · Zbl 1370.74067

[7] B. Benešová, M. Kružík and A. Schlömerkemper, A note on locking materials and gradient polyconvexity, Math. Models Methods Appl. Sci. 28 (2018), no. 12, 2367-2401. · Zbl 1411.49007

[8] A. Braides, Γ-convergence for Beginners, Oxford Lecture Ser. Math. Appl. 22, Oxford University, Oxford, 2005.

[9] R. Choksi, G. Del Piero, I. Fonseca and D. Owen, Structured deformations as energy minimizers in models of fracture and hysteresis, Math. Mech. Solids 4 (1999), no. 3, 321-356. · Zbl 1001.74566

[10] R. Choksi and I. Fonseca, Bulk and interfacial energy densities for structured deformations of continua, Arch. Ration. Mech. Anal. 138 (1997), no. 1, 37-103. · Zbl 0891.73078

[11] F. Christowiak, Homogenization of layered materials with stiff components, PhD thesis, Universität Regensburg, 2018.

[12] F. Christowiak and C. Kreisbeck, Homogenization of layered materials with rigid components in single-slip finite crystal plasticity, Calc. Var. Partial Differential Equations 56 (2017), no. 3, Article ID 75. · Zbl 1375.49015

[13] F. Christowiak and C. Kreisbeck, Asymptotic rigidity of layered structures and its application in homogenization theory,

Arch. Ration. Mech. Anal. (2019), 10.1007/s00205-019-01418-0. · Zbl 1437.49025 · doi:10.1007/s00205-019-01418-0

[14] G. Congedo and I. Tamanini, On the existence of solutions to a problem in multidimensional segmentation, Ann. Inst. H. Poincaré Anal. Non Linéaire 8 (1991), no. 2, 175-195. · Zbl 0729.49003

[15] S. Conti, Relaxation of single-slip single-crystal plasticity with linear hardening, Multiscale Materials Modeling, Fraunhofer IRB, Freiburg (2006), 30-35.

[16] S. Conti, G. Dolzmann and C. Kreisbeck, Asymptotic behavior of crystal plasticity with one slip system in the limit of rigid elasticity, SIAM J. Math. Anal. 43 (2011), no. 5, 2337-2353. · Zbl 1233.35187

[17] S. Conti, G. Dolzmann and C. Kreisbeck, Relaxation of a model in finite plasticity with two slip systems, Math. Models Methods Appl. Sci. 23 (2013), no. 11, 2111-2128. · Zbl 1281.49006

[18] S. Conti and F. Theil, Single-slip elastoplastic microstructures, Arch. Ration. Mech. Anal. 178 (2005), no. 1, 125-148. · Zbl 1076.74017

[19] G. Crasta and V. De Cicco, A chain rule formula in the space BV and applications to conservation laws, SIAM J. Math. Anal. 43 (2011), no. 1, 430-456. · Zbl 1229.26020

[20] G. Dal Maso, An Introduction to Γ-convergence, Progr. Nonlinear Differential Equations Appl. 8, Birkhäuser, Boston, 1993. · Zbl 0816.49001

[21] G. Dal Maso, I. Fonseca, G. Leoni and M. Morini, Higher-order quasiconvexity reduces to quasiconvexity, Arch. Ration. Mech. Anal. 171 (2004), no. 1, 55-81. · Zbl 1082.49017

[22] E. Davoli and G. A. Francfort, A critical revisiting of finite elasto-plasticity, SIAM J. Math. Anal. 47 (2015), no. 1, 526-565. · Zbl 1317.74022

[23] E. Davoli and M. Friedrich, Two-well rigidity and multidimensional sharp-interface limits for solid-solid phase transitions, preprint (2018), https://arxiv.org/abs/1810.06298.

[24] G. Del Piero and D. R. Owen, Structured deformations of continua, Arch. Ration. Mech. Anal. 124 (1993), no. 2, 99-155. · Zbl 0795.73005

[25] R. Ferreira and I. Fonseca, Characterization of the multiscale limit associated with bounded sequences in BV, J. Convex Anal. 19 (2012), no. 2, 403-452. · Zbl 1254.28009

[26] I. Fonseca, G. Leoni and J. Malý, Weak continuity and lower semicontinuity results for determinants, Arch. Ration. Mech. Anal. 178 (2005), no. 3, 411-448. · Zbl 1081.49013

[27] M. Friedrich and M. Kružík, On the passage from nonlinear to linearized viscoelasticity, SIAM J. Math. Anal. 50 (2018), no. 4, 4426-4456. · Zbl 1393.74019

[28] M. Giaquinta and D. Mucci, Maps of bounded variation with values into a manifold: total variation and relaxed energy, Pure Appl. Math. Q. 3 (2007) no. 2, 513-538. · Zbl 1347.49076

[29] D. Grandi and U. Stefanelli, Finite plasticity in P^{\top}P. Part I: Constitutive model, Contin. Mech. Thermodyn. 29 (2017), no. 1, 97-116. · Zbl 1365.74035

[30] D. Grandi and U. Stefanelli, Finite plasticity in P^{\mathsf{T}}P. Part II: Quasi-static evolution and linearization, SIAM J. Math. Anal. 49 (2017), no. 2, 1356-1384. · Zbl 1367.49006

[31] R. Hill, The Mathematical Theory of Plasticity, Clarendon Press, Oxford, 1950. · Zbl 0041.10802

[32] D. Idczak, The generalization of the Du Bois-Reymond lemma for functions of two variables to the case of partial derivatives of any order, Topology in Nonlinear Analysis (Warsaw 1994), Banach Center Publ. 35, Polish Academy of Sciences, Warsaw (1996), 221-236. · Zbl 0868.49015

[33] E. H. Lee, Elastic-plastic deformation at finite strains, J. Appl. Mech. 36 (1969), 1-6. · Zbl 0179.55603

[34] A. Mielke, Finite elastoplasticity Lie groups and geodesics on {\rm SL}(d), Geometry, Mechanics, and Dynamics, Springer, New York (2002), 61-90. · Zbl 1146.74309

[35] A. Mielke, Energetic formulation of multiplicative elasto-plasticity using dissipation distances, Contin. Mech. Thermodyn. 15 (2003), no. 4, 351-382. · Zbl 1068.74522

[36] A. Mielke and T. Roubíček, Rate-independent elastoplasticity at finite strains and its numerical approximation, Math. Models Methods Appl. Sci. 26 (2016), no. 12, 2203-2236. · Zbl 1349.35371

[37] P. M. Naghdi, A critical review of the state of finite plasticity, Z. Angew. Math. Phys. 41 (1990), no. 3, 315-394. · Zbl 0712.73032

[38] P. Podio-Guidugli, Contact interactions, stress, and material symmetry, for nonsimple elastic materials, Theor. Appl. Mech. (Belgrade) 28-29 (2002), 261-276. · Zbl 1085.74004

[39] R. A. Toupin, Elastic materials with couple-stresses, Arch. Ration. Mech. Anal. 11 (1962), 385-414. · Zbl 0112.16805

[40] R. A. Toupin, Theories of elasticity with couple-stress, Arch. Ration. Mech. Anal. 17 (1964), 85-112. · Zbl 0131.22001