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Stochastic effects of waves on currents in the ocean mixed layer. (English) Zbl 1466.76013

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Summary: This paper introduces an energy-preserving stochastic model for studying wave effects on currents in the ocean mixing layer. The model is called stochastic forcing by Lie transport (SFLT). The SFLT model is derived here from a stochastic constrained variational principle, so it has a Kelvin circulation theorem. The examples of SFLT given here treat 3D Euler fluid flow, rotating shallow water dynamics, and the Euler-Boussinesq equations. In each example, one sees the effect of stochastic Stokes drift and material entrainment in the generation of fluid circulation. We also present an Eulerian averaged SFLT model based on decomposing the Eulerian solutions of the energy-conserving SFLT model into sums of their expectations and fluctuations.

MSC:

76B15 Water waves, gravity waves; dispersion and scattering, nonlinear interaction
86A05 Hydrology, hydrography, oceanography
76E20 Stability and instability of geophysical and astrophysical flows
76F25 Turbulent transport, mixing

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[70] In taking these averages over the fast behavior of the dynamics, one must also deal properly with any resonances that would occur. However, the effects of resonances will be neglected in our discussion here.

[71] See Ref. 10 for the corresponding result for the finite-dimensional Euler-Lagrange equation in the absence of symmetry.

[72] Advected quantities are also known as order parameters in condensed matter physics.

[73] Note that we can choose separate (uncorrelated) Brownian motions in the m and a equations in (B6) by choosing $h_i(m, a) = h_i^m(m) + h_i^a(a)$ for the Stratonovich noise, $\circ d W_t^i$. The choice of Stratonovich noise enables the standard calculus chain rule and product rule to be used for the operations of differentiation and integration by parts, respectively, in which variational principles are defined.