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Dynamical properties of feedback signalling in B lymphopoiesis: a mathematical modelling approach. (English) [Zbl 1465.92009]

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Summary: Haematopoiesis is the process of generation of blood cells. Lymphopoiesis generates lymphocytes, the cells in charge of the adaptive immune response. Disruptions of this process are associated with diseases like leukaemia, which is especially incident in children. The characteristics of self-regulation of this process make them suitable for a mathematical study.

In this paper we develop mathematical models of lymphopoiesis using currently available data. We do this by drawing inspiration from existing structured models of cell lineage development and integrating them with paediatric bone marrow data, with special focus on regulatory mechanisms. A formal analysis of the models is carried out, giving steady states and their stability conditions. We use this analysis to obtain biologically relevant regions of the parameter space and to understand the dynamical behaviour of B-cell renovation. Finally, we use numerical simulations to obtain further insight into the influence of proliferation and maturation rates on the reconstitution of the cells in the B line. We conclude that a model including feedback regulation of cell proliferation represents a biologically plausible depiction for B-cell reconstitution in bone marrow. Research into haematological disorders could benefit from a precise dynamical description of B lymphopoiesis.

MSC:

92C15 Developmental biology, pattern formation

34C11 Growth and boundedness of solutions to ordinary differential equations

34D20 Stability of solutions to ordinary differential equations

Keywords:

mathematical medicine; haematopoiesis; mathematical modelling; lymphopoiesis

Software:

flowPeaks; Bioconductor; flowCore

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