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Nonlinear instability for the surface quasi-geostrophic equation in the supercritical regime.

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Summary: We consider the forced surface quasi-geostrophic equation with supercritical dissipation. We show that linear instability for steady state solutions leads to their nonlinear instability. When the dissipation is given by a fractional Laplacian, the nonlinear instability is expressed in terms of the scaling invariant norm, while we establish stronger instability claims in the setting of logarithmically supercritical dissipation. A key tool in treating the logarithmically supercritical setting is a global well-posedness result for the forced equation, which we prove by adapting and extending recent work related to nonlinear maximum principles. We believe that our proof of global well-posedness is of independent interest, to our knowledge giving the first large-data supercritical result with sharp regularity assumptions on the forcing term.

MSC:

76E20 Stability and instability of geophysical and astrophysical flows
76E30 Nonlinear effects in hydrodynamic stability
76M45 Asymptotic methods, singular perturbations applied to problems in fluid mechanics

Keywords:

fractional Laplacian; dissipation; well-posedness; scaling invariant norm; force term asymptotics; logarithmic supercriticality

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