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Regularity results for two standard models in elasto-perfect-plasticity theory with hardening. (English) Zbl 1465.74030

J. Convex Anal. 28, No. 2, 395-428 (2021).

Summary: We consider two most studied standard models in the theory of elasto-plasticity with hardening in arbitrary dimension $d \geq 2$, namely, the kinematic hardening and the isotropic hardening problem. While the existence and uniqueness of the solution is very well known, the optimal regularity up to the boundary remains an open problem. Here, we show that in the interior we have Sobolev regularity for the stress and hardening while for their time derivatives we have the "half" derivative with the spatial and time variable. This was well known for the limiting problem but we show that these estimates are uniform and independent of the order of approximation. The main novelty consist of estimates near the boundary. We show that for the stress and the hardening parameter, we control tangential derivative in the Lebesgue space L^2, and for time derivative of the stress and the hardening we control the "half" time derivative and also spatial tangential derivative. Last, for the normal derivative, we show that the stress and the hardening have the $3/5$ derivative with respect to the normal and for the time derivative of the stress and the hardening we show they have the $1/5$ derivative with respect to the normal direction, provided we consider the kinematic hardening or near the Dirichlet boundary. These estimates are independent of the dimension. In case, we consider the isotropic hardening near the Neumann boundary we shall obtain $W^{\alpha,2}$ regularity for the stress and the hardening with some $\alpha > 1/2$ depending on the dimension and $W^{\beta,2}$ with some $\beta > 1/6$ for the time derivative of the stress and the hardening. Finally, in case of kinematic hardening the same regularity estimate holds true also for the velocity gradient.

MSC:

74C05 Small-strain, rate-independent theories of plasticity (including rigid-plastic and elasto-plastic materials)

74G40 Regularity of solutions of equilibrium problems in solid mechanics

35Q74 PDEs in connection with mechanics of deformable solids

Keywords:

elasto-perfect-plasticity; hardening; Cauchy stress; boundary regularity; fractional regularity; Lebesgues space

Full Text: arXiv Link

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