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**Variational formulations of steady rotational equatorial waves.** (English) Zbl 1464.76203  
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Summary: When the vorticity is monotone with depth, we present a variational formulation for steady periodic water waves of the equatorial flow in the  $f$ -plane approximation, and show that the governing equations for this motion can be obtained by studying variations of a suitable energy functional  $\mathcal{H}$  in terms of the stream function and the thermocline. We also compute the second variation of the constrained energy functional, which is related to the linear stability of steady water waves.

**MSC:**

**76U60** Geophysical flows

**76B15** Water waves, gravity waves; dispersion and scattering, nonlinear interaction

**76M30** Variational methods applied to problems in fluid mechanics

**76E20** Stability and instability of geophysical and astrophysical flows

**35Q35** PDEs in connection with fluid mechanics

Cited in 4 Documents

**Keywords:**

periodic water wave; equatorial vorticity flow; energy functional; linear stability

**Full Text:** [DOI](#)

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