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Plane contact problem on indentation of a flat punch into a transversely-isotropic half-plane with functionally graded transversely-isotropic coating. (English) Zbl 1464.74130

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Summary: Plane contact problem of the theory of elasticity on indentation of a non-deformable punch with a flat base into an elastic transversely-isotropic half-plane with a transversely-isotropic functionally graded coating is considered. Elastic moduli of the coating vary with depth according to arbitrary functions. An approximated analytical solution effective for a whole range of geometrical parameter (relative layer thickness) of the problem is constructed. Some properties of the contact normal pressure under the punch are obtained analytically and illustrated by the numerical examples for a transversely-isotropic homogeneous and functionally graded coatings with different types of variation of elastic moduli with depth. The distinctions in distribution of contact normal pressure for homogeneous and functionally graded materials, coated and non-coated bodies are studied analytically and numerically.

MSC:

74M15 Contact in solid mechanics
74E30 Composite and mixture properties

Keywords:

contact problem; elasticity; indentation; functionally-graded material; coating; asymptotic methods; semi-analytical solution; plane contact; flat punch

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