

**Nagode, Marko; Klemenc, Jernej; Oman, Simon; Šeruga, Domen**

**A closed-form solution for temperature-dependent elastoplastic problems using the Prandtl operator approach.** (English) [Zbl 1464.74032](#)

Commun. Nonlinear Sci. Numer. Simul. 99, Article ID 105839, 24 p. (2021).

Summary: Finite element simulations of the temperature-dependent stress-strain response in the elastoplastic region of a material usually involve incremental procedures based on the Newton-Raphson iterative scheme. Although essential to obtaining the correct result, iterations inherently extend the computational time of the simulations. In order to increase the computational effectiveness of such finite element simulations, a novel solution technique is presented here, which introduces a closed-form determination of the elastoplastic stress-strain response using the Prandtl operator approach. Using this solution, the iterative procedure is no longer required. The positions of the tensile-compressive and shear meridians of the Haigh-Westergaard coordinate space are first conveniently modified, which then enables the configuration of coordinate-independent play operators. These play operators connect the stress and the strain tensors in a unique closed-form solution that significantly increases the computational power of the simulations, while retaining both the vigorous stability of the procedure and the high accuracy of the results. The method is successfully validated on several load cases, consisting of variable tensile, shear and combined thermomechanical load histories. Limitations of the current version of the approach, that are a part of on-going research, include extremely low values of the third deviatoric strain-invariant increments, in which the directions of movement of the yield surfaces can be changed. Furthermore, the discretisation of the cyclic stress-strain curves plays an important role. The optimal positions of the yield strains are hence another important issue for future studies. Additionally, the consistent material Jacobian results in an unsymmetric form in e.g. Abaqus when engineering shear strains are provided in the simulations and thus the computational power is not fully used. Nevertheless, the results using the Prandtl operator approach, when compared to the results obtained using the conventional, Besseling material model, show excellent agreement, while substantially reducing the computational time by up to 45%.

**MSC:**

**74C05** Small-strain, rate-independent theories of plasticity (including rigid-plastic and elasto-plastic materials)

**74S05** Finite element methods applied to problems in solid mechanics

**74F05** Thermal effects in solid mechanics

**Keywords:**

[Prandtl operator](#); [finite element method](#); [thermomechanical loading](#); [uniaxial load](#); [stress-strain response](#)

**Full Text:** [DOI](#)

**References:**

- [1] Yoosefian, A.; Golmakani, M.; Sadeghian, M., Nonlinear bending of functionally graded sandwich plates under mechanical and thermal load, Commun Nonlinear Sci Numer Simul, 84, 105161 (2020) · [Zbl 1450.74026](#)
- [2] Liu, G.; Winwood, S.; Rhodes, K.; Biroasca, S., The effects of grain size, dendritic structure and crystallographic orientation on fatigue crack propagation in IN713C nickel-based superalloy, Int J Plast, 125, 150-168 (2020)
- [3] Zhang, Q.-L.; Hu, L.; Hu, C.; Wu, H.-G., Low-cycle fatigue issue of steel spiral cases in pumped-storage power plants under China's and US's design philosophies: a comparative numerical case study, Int J Press Vessels Pip, 172, 134-144 (2019)
- [4] Zhao, L.; Xu, L.; Han, Y.; Jing, H.; Gao, Z., Modelling creep-fatigue behaviours using a modified combined kinematic and isotropic hardening model considering the damage accumulation, Int J Mech Sci, 161-162, 105016 (2019)
- [5] Khazaei, M.; Khadem, S.; Moslemi, A.; Abdollahi, A., Vibration mitigation of a pipe conveying fluid with a passive geometrically nonlinear absorber: a tuning optimal design, Commun Nonlinear Sci Numer Simul, 91, 105439 (2020) · [Zbl 1453.74038](#)
- [6] Šeruga, D.; Hansenne, E.; Haesen, V.; Nagode, M., Durability prediction of EN 1.4512 exhaust mufflers under thermomechanical loading, Int J Mech Sci, 84, 199-207 (2014)
- [7] Callaghan, M.; Humphries, S.; Law, M.; Ho, M.; Yan, K.; Yeung, W., Specimen-size dependency and modelling of energy evolution during high-temperature low-cycle fatigue of pressure vessel steel, Scr Mater, 65, 4, 308-311 (2011)

- [8] Pandey, R.; Shukla, K.; Jain, A., Thermoelastic stability analysis of laminated composite plates: an analytical approach, *Commun Nonlinear Sci Numer Simul*, 14, 4, 1679-1699 (2009) · [Zbl 1221.74021](#)
- [9] fang Xie, X.; Jiang, W.; Chen, J.; Zhang, X.; Tu, S.-T., Cyclic hardening/softening behavior of 316L stainless steel at elevated temperature including strain-rate and strain-range dependence: experimental and damage-coupled constitutive modeling, *Int J Plast*, 114, 196-214 (2019)
- [10] Li, J.; Romero, I.; Segurado, J., Development of a thermo-mechanically coupled crystal plasticity modeling framework: application to polycrystalline homogenization, *Int J Plast*, 119, 313-330 (2019)
- [11] Šeruga, D.; Nagode, M., A new approach to finite element modelling of cyclic thermomechanical stress-strain responses, *Int J Mech Sci*, 164, 105139 (2019)
- [12] Brepols, T.; Wulfinghoff, S.; Reese, S., A gradient-extended two-surface damage-plasticity model for large deformations, *Int J Plast*, 129, 102635 (2020)
- [13] Yilmaz, O. F.; Ozcelik, G.; Yeni, F. B., Lean holistic fuzzy methodology employing cross-functional worker teams for new product development projects: a real case study from high-tech industry, *Eur J Oper Res*, 282, 3, 989-1010 (2020) · [Zbl 1431.90092](#)
- [14] Institutional Perspectives on Supply Chain Management
- [15] Ottosen, N.; Ristinmaa, M., *The mechanics of constitutive modeling* (2005), Elsevier Science
- [16] Chen, W.-F.; Han, D.-J., *Plasticity for structural engineers* (2007), J. Ross Publishing · [Zbl 0666.73010](#)
- [17] Lode, W., Versuche ber den Einflu der mittleren Hauptspannung auf das Flieen der Metalle Eisen, Kupfer und Nickel, *Z Phys*, 36, 11, 913-939 (1926)
- [18] Paliwal, B.; Hammi, Y.; Chandler, M.; Moser, R.; Horstemeyer, M., A three-invariant cap-viscoplastic rate-dependent-damage model for cementitious materials with return mapping integration in Haigh-Westergaard coordinate space, *Int J Solids Struct*, 182-183, 77-99 (2020)
- [19] Piccolroaz, A.; Bigoni, D., Yield criteria for quasibrittle and frictional materials: a generalization to surfaces with corners, *Int J Solids Struct*, 46, 20, 3587-3596 (2009) · [Zbl 1183.74032](#)
- [20] Zarkovic, D.; Jovanovic, D.; Vukobratovic, V.; Brujic, Z., Convergence improvement in computation of strain-softening solids by the arc-length method, *Finite Elem Anal Des*, 164, 55-68 (2019)
- [21] Lee, Y.; Ghosh, J., The significance of J3 to the prediction of shear bands, *Int J Plast*, 12, 9, 1179-1197 (1996) · [Zbl 0887.73022](#)
- [22] Driemeier, L.; Baroncini Proenca, S. P.; Alves, M., A contribution to the numerical nonlinear analysis of three-dimensional truss systems considering large strains, damage and plasticity, *Commun Nonlinear Sci Numer Simul*, 10, 5, 515-535 (2005) · [Zbl 1068.74074](#)
- [23] Hill, R., *The mathematical theory of plasticity* (1950), Oxford University Press · [Zbl 0041.10802](#)
- [24] Melan, E., Zur plastizität des räumlichen kontinuums, *Ing Arch*, 9, 116-126 (1938) · [Zbl 64.0840.01](#)
- [25] Prager, D. W., The theory of plasticity: a survey of recent achievements, *Proc Inst Mech Eng*, 169, 1, 41-57 (1955)
- [26] Lee, J.; Bong, H. J.; Kim, S.-J.; Lee, M.-G.; Kim, D., An enhanced distortional-hardening-based constitutive model for hexagonal close-packed metals: application to az31b magnesium alloy sheets at elevated temperatures, *Int J Plast*, 126, 102618 (2020)
- [27] Qi, Z.; Hu, N.; Li, G.; Zeng, D.; Su, X., Constitutive modeling for the elastic-viscoplastic behavior of high density polyethylene under cyclic loading, *Int J Plast*, 113, 125-144 (2019)
- [28] Xing, R.; Yu, D.; Shi, S.; Chen, X., Cyclic deformation of 316L stainless steel and constitutive modeling under non-proportional variable loading path, *Int J Plast*, 120, 127-146 (2019)
- [29] Luo, C.; Zeng, W.; Sun, J.; Yuan, H., Plasticity modeling for a metastable austenitic stainless steel with strain-induced martensitic transformation under cyclic loading conditions, *Mater Sci Eng*, 775, 138961 (2020)
- [30] Madrigal, C.; Navarro, A.; Chaves, V., A simplified plasticity model for multiaxial non-proportional cyclic loading, *Theor Appl Fract Mech*, 103, 102247 (2019)
- [31] Petkov, M. P.; Hu, J.; Tarleton, E.; Cocks, A. C., Comparison of self-consistent and crystal plasticity fe approaches for modelling the high-temperature deformation of 316h austenitic stainless steel, *Int J Solids Struct*, 171, 54-80 (2019)
- [32] Rodas, E. A.E.; Neu, R. W., Crystal viscoplasticity model for the creep-fatigue interactions in single-crystal ni-base superalloy CMSX-8, *Int J Plast*, 100, 14-33 (2018)
- [33] Cruzado, A.; Lucarini, S.; LLorca, J.; Segurado, J., Microstructure-based fatigue life model of metallic alloys with bilinear coffin-manson behavior, *Int J Fatigue*, 107, 40-48 (2018)
- [34] Farooq, H.; Cailletaud, G.; Forest, S.; Ryckelynck, D., Crystal plasticity modeling of the cyclic behavior of polycrystalline aggregates under non-symmetric uniaxial loading: global and local analyses, *Int J Plast*, 126, 102619 (2020)
- [35] Haibach, E., *Betriebsfestigkeit - verfahren und daten zur bauteilberechnung* (2006), Springer-Verlag Berlin Heidelberg
- [36] Okorokov, V.; Gorash, Y.; Mackenzie, D.; van Rijswijk, R., New formulation of nonlinear kinematic hardening model, part II: cyclic hardening/softening and ratcheting, *Int J Plast*, 122, 244-267 (2019)
- [37] Hartmann, S.; Bier, W., High-order time integration applied to metal powder plasticity, *Int J Plast*, 24, 1, 17-54 (2008) · [Zbl 1290.74042](#)
- [38] Nguyen, L. H.; Schilling, D., The multiscale finite element method for nonlinear continuum localization problems at full fine-scale fidelity, illustrated through phase-field fracture and plasticity, *J Comput Phys*, 396, 129-160 (2019) · [Zbl 1452.74110](#)

- [39] Gosciniak, I.; Gdawiec, K., Control of dynamics of the modified Newton-Raphson algorithm, *Commun Nonlinear Sci Numer Simul*, 67, 76-99 (2019) · [Zbl 07263872](#)
- [40] Krabbenhøft, K., *Basic Computational Plasticity*, Tech. Rep. (2002), Technical University of Denmark, Copenhagen
- [41] Nagode, M.; Zingsheim, F., An online algorithm for temperature influenced fatigue life estimation: strain-life approach, *Int J Fatigue*, 26, 2, 155-161 (2004) · [Zbl 1060.74608](#)
- [42] Nagode, M.; Fajdiga, M., Temperature-stress-strain trajectory modeling during thermo-mechanical fatigue, *Fatigue Fract Eng Mater Struct*, 29, 175-182 (2005)
- [43] Nagode, M.; Fajdiga, M., Thermo-mechanical modelling of stochastic stress-strain states, *J Mech Eng*, 52, 74-84 (2006)
- [44] Nagode, M.; Fajdiga, M., Coupled elastoplasticity and viscoplasticity under thermomechanical loading, *Fatigue Fract Eng Mater Struct*, 30, 6, 510-519 (2007)
- [45] Nagode, M.; Hack, M.; Fajdiga, M., Low cycle thermo-mechanical fatigue: damage operator approach, *Fatigue Fract Eng Mater Struct*, 33, 149-160 (2009)
- [46] Nagode, M.; Laengler, F.; Hack, M., A time-dependent damage operator approach to thermo-mechanical fatigue of Ni-resist D-5S, *Int J Fatigue*, 33, 5, 692-699 (2011)
- [47] Šolinc, U.; Klemenc, J.; Nagode, M.; Šeruga, D., A direct approach to modelling the complex response of magnesium AZ31 alloy sheets to variable strain amplitude loading using Prandtl-Ishlinskii operators, *Int J Fatigue*, 127, 291-304 (2019)
- [48] Nagode, M.; Laengler, F.; Hack, M., Damage operator based lifetime calculation under thermo-mechanical fatigue for application on Ni-resist D-5S turbine housing of turbocharger, *Eng Fail Anal*, 18, 6, 1565-1575 (2011)
- [49] Nagode, M.; Šeruga, D.; Hack, M.; Hansenne, E., Damage operator-based lifetime calculation under thermomechanical fatigue and creep for application on uginox F12T EN 1.4512 exhaust downpipes, *Strain*, 48, 3, 198-207 (2012)
- [50] Nagode, M.; Gosar, A.; Hack, M.; Hansenne, E.; Šeruga, D., A review of thermomechanical fatigue damage calculations with the damage operator approach, LCF8 - eighth international conference on low cycle fatigue (2016)
- [51] 33rd Danubia Adria Symposium on Advances in Experimental Mechanics, 20-23 September 2016.
- [52] Bartošák, M.; Španiel, M.; Doubrava, K., Thermo-mechanical fatigue of SiMo 4.06 turbocharger turbine housing: damage operator approach, *Eng Fail Anal*, 105, 736-755 (2019)
- [53] Besseling, J., A theory of elastic, plastic and creep deformations of an initially isotropic material, *J Appl Mech*, 25, 529-536 (1958) · [Zbl 0084.20501](#)
- [54] Šeruga, D.; Nagode, M., Comparative analysis of optimisation methods for linking material parameters of exponential and power models: an application to cyclic stress-strain curves of ferritic stainless steel, *Proc Inst Mech Eng Part L*, 233, 9, 1802-1813 (2019)
- [55] Šeruga, D.; Nagode, M.; Klemenc, J., Eliminating friction between flat specimens and an antibuckling support during cyclic tests using a simple sensor, *Meas Sci Technol*, 30, 9, 095102 (2019)
- [56] Simo, J.; Taylor, R., Consistent tangent operators for rate-independent elastoplasticity, *Comput Methods Appl Mech Eng*, 48, 1, 101-118 (1985) · [Zbl 0535.73025](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.