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Sharp rigidity estimates for incompatible fields as a consequence of the Bourgain Brezis div-curl result. (Estimées de rigidité pour les champs incompatibles comme conséquence du résultat div-rot de Bourgain et Brezis.) (English. French summary) Zbl 1464.49033
C. R., Math., Acad. Sci. Paris 359, No. 2, 155-160 (2021).

The authors derive a sharp rigidity estimate and a sharp Korn's inequality for matrix-valued fields. The main results follow:

Given an open, bounded, connected and Lipschitz set $\Omega \subset \mathbb{R}^n$, $n \geq 2$, there exists $C > 0$ such that for every $\beta \in L^1(\Omega; \mathbb{R}^{n \times n})$ with $\text{Curl } \beta \in \mathcal{M}(\Omega; \mathbb{R}^{n \times n \times n})$, there exist a rotation $R \in \text{SO}(n)$ such that

$$\|\beta - R\|_{L^{1^*}(\Omega)} \leq C(\|\text{dist}(\beta, \text{SO}(n))\| + |\text{Curl } \beta|(\Omega)).$$

and an antisymmetric matrix A such that

$$\|\beta - A\|_{L^{1^*}(\Omega)} \leq C(\|\beta + \beta^T\|_{L^{1^*}(\Omega)} + |\text{Curl } \beta|(\Omega))$$

where $\mathcal{M}(\Omega; \mathbb{R}^{n \times n \times n})$ is the set of Radon measures on Ω with values in $\mathbb{R}^{n \times n \times n}$ and $1^* = n/(n-1)$ is the Sobolev conjugate exponent.

Reviewer: Igor Bock (Bratislava)

MSC:

- 49Q20** Variational problems in a geometric measure-theoretic setting
- 74C15** Large-strain, rate-independent theories of plasticity (including nonlinear plasticity)
- 53C24** Rigidity results

Cited in 4 Documents

Keywords:

rigidity estimates; div-curl systems; Korn inequality; plasticity; incompatible fields; Hodge decomposition

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