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On the automorphism groups of vertex-transitive Cayley digraphs of monoids. (English)

Zbl 1464.05180
J. Algebr. Comb. 53, No. 1, 227-251 (2021).

Summary: *L. Babai* and *C. D. Godsil* [Eur. J. Comb. 3, 9–15 (1982; Zbl 0483.05033)] conjectured that almost all Cayley digraphs are digraphical regular representations. *M. Xu* [Discrete Math. 182, No. 1–3, 309–319 (1998; Zbl 0887.05025)] conjectured that almost all Cayley digraphs are normal (i.e., G_L is a normal subgroup of the automorphism group of $\mathrm{Cay}(G, C)$). Finally, in 1994, Praeger and Mckay conjectured that almost all undirected vertex-transitive graphs are Cayley graphs of groups. In this paper, first we present the variants of these conjectures for Cayley digraphs of monoids and we determine when the variant of Babai and Godsil's conjecture [loc. cit.] is equivalent to the variant of Xu's conjecture. Then, as a special consequence of our results, we conclude that Xu's conjecture [loc. cit.] is equivalent to Babai and Godsil's conjecture. On the other hand, we give affirmative answer to the variant of Praeger and Mckay's conjecture and we prove that a Cayley digraph of a monoid is vertex-transitive if and only if it is isomorphic to a Cayley digraph of a group. Finally, we use this characterization to give an affirmative answer to a question raised by *A. V. Kelarev* and *C. E. Praeger* [Eur. J. Comb. 24, No. 1, 59–72 (2003; Zbl 1011.05027)] about vertex-transitivity of Cayley digraphs of monoids. Also using this characterization, we explicitly determine the automorphism groups of vertex-transitive Cayley digraphs of monoids.

MSC:

05C25 Graphs and abstract algebra (groups, rings, fields, etc.)
05C20 Directed graphs (digraphs), tournaments
05C62 Graph representations (geometric and intersection representations, etc.)
06A15 Galois correspondences, closure operators (in relation to ordered sets)
20B25 Finite automorphism groups of algebraic, geometric, or combinatorial structures

Keywords:

normal Cayley digraphs; digraphical regular representation; closure operators

Full Text: DOI

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