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**Implication operators generating pairs of weak negations and their algebraic structure.**

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Summary: Negations operators have been developed and applied in many fields such as image processing, decision making, mathematical morphology, fuzzy logic, etc. One of the most effective non-monotonic operators are weak negations. This paper studies the algebraic structure and the characterization of the adjoint triples and Galois implication pairs which provides a fixed pair of weak negations. The obtained results allow the user to select the best conjunctive and implications associated with the most suitable negation to be used in the computations of the problem to be solved.

**MSC:**

**03G10** Logical aspects of lattices and related structures

**03B52** Fuzzy logic; logic of vagueness

**06A15** Galois correspondences, closure operators (in relation to ordered sets)

**Keywords:**

fuzzy sets; adjoint triples; negation operators; pair of weak negations

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