Bingham, Nicholas H.; Ostaszewski, Adam J.
The Steinhaus-Weil property. I: Subcontinuity and amenability. (English)
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Summary: The Steinhaus-Weil theorem that concerns us here is the simple, or classical, ‘interior-points’ property – that in a Polish topological group a nonnegligible set $B$ has the identity as an interior point of $B^{-1}B$. There are various converses; the one that mainly concerns us is due to Simmons and Mospan. Here the group is locally compact, so we have a Haar reference measure $\eta$. The Simmons-Mospan theorem states that a (regular Borel) measure has such a Steinhaus-Weil property if and only if it is absolutely continuous with respect to the Haar measure. This the first of four companion papers (we refer to the others as II [“The Steinhaus-Weil property. II: The Simmons-Mospan converse”, ibid. (to appear)], III [“The Steinhaus-Weil property. III: Weil topologies”], and IV [“The Steinhaus-Weil property. IV: Other interior-point properties”]). Here (Propositions 1.1–1.7 and Theorems 1.1–1.4) we exploit the connection between the interior-points property and a selective form of infinitesimal invariance afforded by a certain family of selective reference measures $\sigma$, drawing on Solecki’s amenability at 1 (and using Fuller’s notion of subcontinuity).

In II, we turn to a converse of the Steinhaus-Weil theorem, the Simmons-Mospan theorem, and related results. In III, we discuss Weil topologies, linking the topological group-theoretic and measure-theoretic aspects. We close in IV with some other interior-point results related to the Steinhaus-Weil theorem.

MSC:
22A10 Analysis on general topological groups
43A05 Measures on groups and semigroups, etc.
28C10 Set functions and measures on topological groups or semigroups, Haar measures, invariant measures

Keywords:
Steinhaus-Weil property; amenability at 1; measure subcontinuity; Simmons-Mospan theorem; selective measure; interior-points property; Haar measure; left Haar null

Full Text: DOI

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