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Generalized Chebyshev polynomials. (English) Zbl 1463.11086

Summary: Let \( h(x) \) be a non constant polynomial with rational coefficients. Our aim is to introduce the \( h(x) \)-Chebyshev polynomials of the first and second kind \( T_n \) and \( U_n \). We show that they are in a \( \mathbb{Q} \)-vectorial subspace \( E_n(x) \) of \( \mathbb{Q}[x] \) of dimension \( n \). We establish that the polynomial sequences \( (h^k T_{n-k})_k \) and \( (h^k U_{n-k})_k \), \( 0 \leq k \leq n - 1 \) are two bases of \( E_n(x) \) for which \( T_n \) and \( U_n \) admit remarkable integer coordinates.

MSC:
11B83 Special sequences and polynomials
15A03 Vector spaces, linear dependence, rank, lineability
11B39 Fibonacci and Lucas numbers and polynomials and generalizations
08A40 Operations and polynomials in algebraic structures, primal algebras

Keywords:
Chebyshev polynomials; integer coordinates; polynomial bases

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