

**Drouot, Alexis**

**Microlocal analysis of the bulk-edge correspondence.** (English) Zbl 1462.81214  
*Commun. Math. Phys.* 383, No. 3, 2069-2112 (2021).

Summary: The bulk-edge correspondence predicts that interfaces between topological insulators support robust currents. We prove this principle for PDEs that are periodic away from an interface. Our approach relies on semiclassical methods. It suggests novel perspectives for the analysis of topologically protected transport.

**MSC:**

- 81V70 Many-body theory; quantum Hall effect
- 35J10 Schrödinger operator, Schrödinger equation
- 81Q10 Selfadjoint operator theory in quantum theory, including spectral analysis
- 82B20 Lattice systems (Ising, dimer, Potts, etc.) and systems on graphs arising in equilibrium statistical mechanics
- 35Q40 PDEs in connection with quantum mechanics
- 82D20 Statistical mechanics of solids
- 35P05 General topics in linear spectral theory for PDEs
- 81Q20 Semiclassical techniques, including WKB and Maslov methods applied to problems in quantum theory
- 35P20 Asymptotic distributions of eigenvalues in context of PDEs
- 35S05 Pseudodifferential operators as generalizations of partial differential operators
- 46L87 Noncommutative differential geometry
- 19L10 Riemann-Roch theorems, Chern characters

Cited in **3** Documents

**Full Text:** [DOI](#) [arXiv](#)

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