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Synchronization of Kuramoto oscillators: inverse Taylor expansions. (English) Zbl 1462.34081
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This comprehensive paper on Kuramoto model starts from the standard heterogeneous Kuramoto model

$$\dot{\theta}_i = \omega_i - \sum_{j=1}^n a_{ij} \sin(\theta_i - \theta_j), \quad i = \overline{1, n}$$

and considers $\{\mathbb{T}^n\}$ to be its state space (due to the periodic nonlinearity \sin). The essence of the approach is the analysis of the synchronization manifold and the convergence analysis using Taylor series expansions around it.

Reviewer: Daniela Danciu (Craiova)

MSC:

- [34D06](#) Synchronization of solutions to ordinary differential equations
- [34C15](#) Nonlinear oscillations and coupled oscillators for ordinary differential equations
- [34A25](#) Analytical theory of ordinary differential equations: series, transformations, transforms, operational calculus, etc.
- [34D05](#) Asymptotic properties of solutions to ordinary differential equations

Keywords:

[oscillators](#); [frequency synchronization](#); [synchronization manifold](#); [Taylor series](#); [power networks](#)

Software:

[MATPOWER](#)

Full Text: [DOI](#) [arXiv](#)

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