Devin, Lucile

One can begin with author’s abstract: “We generalize current known distribution results on Shanks-Rényi prime number races to the case where arbitrarily many residue classes are involved. Our method handles both the classical case that goes back to Chebyshev and function field analogues developed in the recent years. More precisely, let \( \pi(x; q, a) \) be the number of primes up to \( x \) that are congruent to \( a \) modulo \( q \). For a fixed integer \( q \) and distinct invertible congruence classes \( a_0, a_1, \ldots, a_D \), assuming the generalized Riemann Hypothesis and a weak version of the linear independence hypothesis, we show that the set of real \( x \) for which the inequalities \( \pi(x; q, a_0) > \pi(x; q, a_1) > \ldots > \pi(x; q, a_D) \) are simultaneously satisfied admits a logarithmic density.”

In a survey of this paper, the attention is given to results related to the generalized Riemann hypothesis (GRH) and the linear independence (LI) conjecture for the zeros of the Dirichlet \( L \)-functions of the non-principal characters modulo \( q \), and also to a natural translation of the notion of prime number races to irreducible polynomials with coefficients in finite fields.

Auxiliary notions such as the logarithmic density of a set of positive real numbers, the inferior and superior logarithmic density, etc., are recalled.

The present results are proven with explanations and their connections with known investigations are discussed.

Reviewer: Symon Serbenyuk (Kyïv)

MSC:

11K70 Harmonic analysis and almost periodicity in probabilistic number theory
28C15 Set functions and measures on topological spaces (regularity of measures, etc.)
11M26 Nonreal zeros of \( \zeta(s) \) and \( L(s, \chi) \); Riemann and other hypotheses

Keywords:
Chebyshev’s bias; almost-periodic function; regularity of measure

Full Text: DOI

References:
