Given a lattice $L$ of full rank in $n$-dimensional real space, a vector in the lattice is called $i$-sparse if it has no more than $i$ non-zero coordinates. Define the $i$-th successive sparsity level of $L$, denoted $s_i(L)$, to be the minimal $s$ such that the lattice has $s$ linearly independent $i$-sparse vectors. The authors give sufficient conditions for $s_i(L)$ to be smaller than $n$ and give explicit bounds on the sup-norms of the corresponding linearly independent sparse vectors in $L$. They use this result to study virtually rectangular lattices, establishing conditions for the lattice to be virtually rectangular and determining the index of a rectangular sublattice. In the 2-dimensional situation, they show that virtually rectangular lattices in the plane correspond to elliptic curves isogenous to those with real $j$-invariant. They identify planar virtually rectangular lattices in terms of a natural rationality condition of the geodesics on the modular curve carrying the corresponding points.

Reviewer: Steven T. Dougherty (Scranton)

MSC:

11H06 Lattices and convex bodies (number-theoretic aspects)
52C07 Lattices and convex bodies in $n$ dimensions (aspects of discrete geometry)
11G05 Elliptic curves over global fields

Keywords:
lattices; sparse vectors; virtually rectangular lattices; Siegel’s lemma; elliptic curve; $j$-invariant; isogeny; modular curve; geodesics

Full Text: DOI

References:


