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The centre-mode instability of viscoelastic plane Poiseuille flow. (English) Zbl 1461.76025
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Summary: A modal stability analysis shows that plane Poiseuille flow of an Oldroyd-B fluid becomes unstable to a 'centre mode' with phase speed close to the maximum base-flow velocity, $U_{\max}$. The governing dimensionless groups are the Reynolds number $Re = \rho U_{\max} H/\eta$, the elasticity number $E = \lambda\eta/(H^2\rho)$ and the ratio of solvent to solution viscosity $\beta = \eta_s/\eta$; here, λ is the polymer relaxation time, H is the channel half-width and ρ is the fluid density. For experimentally relevant values (e.g. $E \sim 0.1$ and $\beta \sim 0.9$), the critical Reynolds number, Re_c, is around 200, with the associated eigenmodes being spread out across the channel. For $E(1-\beta) \ll 1$, with E fixed, corresponding to strongly elastic dilute polymer solutions, $Re_c \propto (E(1-\beta))^{-3/2}$ and the critical wavenumber $k_c \propto (E(1-\beta))^{-1/2}$. The unstable eigenmode in this limit is confined in a thin layer near the channel centreline. These features are largely analogous to the centre-mode instability in viscoelastic pipe flow [*P. Garg* et al., "Viscoelastic pipe flow is linearly unstable", Phys. Rev. Lett. 121, No. 2, Article ID 024502, 6 p. (2018; doi:10.1103/PhysRevLett.121.024502)], and suggest a universal linear mechanism underlying the onset of turbulence in both channel and pipe flows of sufficiently elastic dilute polymer solutions. Although the centre-mode instability continues down to $\beta \sim 10^{-2}$ for pipe flow, it ceases to exist for $\beta < 0.5$ in channels. Whereas inertia, elasticity and solvent viscous effects are simultaneously required for this instability, a higher viscous threshold is required for channel flow. Further, in the opposite limit of $\beta \to 1$, the centre-mode instability in channel flow continues to exist at $Re \approx 5$, again in contrast to pipe flow where the instability ceases to exist below $Re \approx 63$, regardless of E or β. Our predictions are in reasonable agreement with experimental observations for the onset of turbulence in the flow of polymer solutions through microchannels.

MSC:

76A10 Viscoelastic fluids
76F06 Transition to turbulence

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