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Dissipative boundary conditions and entropic solutions in dynamical perfect plasticity.
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Summary: We prove the well-posedness of a dynamical perfect plasticity model under general assumptions on the stress constraint set and on the reference configuration. The problem is studied by combining both calculus of variations and hyperbolic methods. The hyperbolic point of view enables one to derive a class of dissipative boundary conditions, somehow intermediate between homogeneous Dirichlet and Neumann ones. By using variational methods, we show the existence and uniqueness of solutions. Then we establish the equivalence between the original variational solutions and generalized entropic-dissipative ones, derived from a weak hyperbolic formulation for initial-boundary value Friedrichs' systems with convex constraints.

MSC:

74C05 Small-strain, rate-independent theories of plasticity (including rigid-plastic and elasto-plastic materials)
35Q74 PDEs in connection with mechanics of deformable solids

Keywords:

calculus of variations; hyperbolic conservation law; generalized entropic-dissipative solution; Friedrichs system; existence; uniqueness

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