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**Stability of the anabatic Prandtl slope flow in a stably stratified medium.** (English)

Zbl 1460.76324

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**Summary:** In the Prandtl model for anabatic slope flows, a uniform positive buoyancy flux at the surface drives an upslope flow against a stable background stratification. In the present study, we conduct linear stability analysis of the anabatic slope flow under this model and contrast it against the katabatic case as presented in [the authors, *ibid.* 865, Paper No. R2, 14 p. (2019; Zbl 1429.86005)]. We show that the buoyancy component normal to the sloped surface is responsible for the emergence of stationary longitudinal rolls, whereas a generalised Kelvin-Helmholtz (KH) type of mechanism consisting of shear instability modulated by buoyancy results in a streamwise-travelling mode. In the anabatic case, for slope angles larger than  $9^\circ$  to the horizontal, the travelling KH mode is dominant whereas, at lower inclination angles, the formation of the stationary vortex instability is favoured. The same dynamics holds qualitatively for the katabatic case, but the mode transition appears at slope angles of approximately  $62^\circ$ . For a fixed slope angle and Prandtl number, we demonstrate through asymptotic analysis of linear growth rates that it is possible to devise a classification scheme that demarcates the stability of Prandtl slope flows into distinct regimes based on the dimensionless stratification perturbation number. We verify the existence of the instability modes with the help of direct numerical simulations, and observe close agreements between simulation results and predictions of linear analysis. For slope angle values in the vicinity of the junction point in the instability map, both longitudinal rolls and travelling waves coexist simultaneously and form complex flow structures.

**MSC:**

**76E20** Stability and instability of geophysical and astrophysical flows

**76F45** Stratification effects in turbulence

**76E05** Parallel shear flows in hydrodynamic stability

**Keywords:**

atmospheric flows; stratified flows

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**References:**

- [1] Baines, P. G. & Mitsudera, H. 1994 On the mechanism of shear flow instabilities. *J. Fluid Mech.* 276, 327-342. · Zbl 0889.76019
- [2] Banta, R. M. 1984 Daytime boundary-layer evolution over mountainous terrain. Part 1. Observations of the dry circulations. *Mon. Weath. Rev.* 112 (2), 340-356.
- [3] Beare, R. J., Macvean, M. K., Holtslag, A. A. M., Cuxart, J., Esau, I., Golaz, J.-C., Jimenez, M. A., Khairoutdinov, M., Kosovic, B., Lewellen, D. et al. 2006 An intercomparison of large-eddy simulations of the stable boundary layer. *Boundary-Layer Meteorol.* 118 (2), 247-272.
- [4] Candelier, J., Le Dizès, S. & Millet, C. 2011 Shear instability in a stratified fluid when shear and stratification are not aligned. *J. Fluid Mech.* 685, 191-201. · Zbl 1241.76171
- [5] Candelier, J., Le Dizès, S. & Millet, C. 2012 Inviscid instability of a stably stratified compressible boundary layer on an inclined surface. *J. Fluid Mech.* 694, 524-539. · Zbl 1250.76066
- [6] Carpenter, J. R., Balmforth, N. J. & Lawrence, G. A. 2010 Identifying unstable modes in stratified shear layers. *Phys. Fluids* 22 (5), 054104.
- [7] Carpenter, J. R., Tedford, E. W., Heifetz, E. & Lawrence, G. A. 2011 Instability in stratified shear flow: review of a physical interpretation based on interacting waves. *Appl. Mech. Rev.* 64 (6), 060801.
- [8] Chen, J., Bai, Y. & Le Dizès, S. 2016 Instability of a boundary layer flow on a vertical wall in a stably stratified fluid. *J. Fluid Mech.* 795, 262-277. · Zbl 1359.76108
- [9] Chen, T. S. & Tzuoo, K.-L. 1982 Vortex instability of free convection flow over horizontal and inclined surfaces. *Trans. ASME J. Heat Transfer* 104 (4), 637-643.
- [10] Clever, R. M. & Busse, F. H. 1977 Instabilities of longitudinal convection rolls in an inclined layer. *J. Fluid Mech.* 81 (1),

- [11] Coleman, G. N., Ferziger, J. H. & Spalart, P. R. 1990A numerical study of the turbulent Ekman layer. *J. Fluid Mech.* 213, 313-348.
- [12] Defant, F. 1949a Zur Theorie der Hangwinde, nebst Bemerkungen zur Theorie der Berg- und Talwinde. *Archiv für Meteorologie, Geophysik und Bioklimatologie, Serie A1* (3-4), 421-450.
- [13] Defant, F. 1949b Zur Theorie der Hangwinde, nebst Bemerkungen zur Theorie der Berg- und Talwinde (A theory of slope winds, along with remarks on the theory of mountain winds and valley winds). *Archiv für Meteorologie, Geophysik und Bioklimatologie, Serie A (Theoretical and Applied Climatology)* 1 (3-4), 421-450; (English translation: Whiteman, C. D., and Dreiseitl, E., 1984: *Alpine meteorology: Translations of classic contributions by A. Wagner, E. Ekhart and F. Defant.* PNL-5141/ASCOT-84-3. Pacific Northwest Laboratory, Richland, Washington, 121 pp).
- [14] Deloncle, A., Chomaz, J.-M. & Billant, P. 2007 Three-dimensional stability of a horizontally sheared flow in a stably stratified fluid. *J. Fluid Mech.* 570, 297-305. · [Zbl 1105.76025](#)
- [15] Drazin, P. G. & Reid, W. H. 2004 *Hydrodynamic Stability*, 2nd edn. Cambridge University Press.
- [16] Eaves, T. S. & Balmforth, N. J. 2019 Instability of sheared density interfaces. *J. Fluid Mech.* 860, 145-171. · [Zbl 1415.86006](#)
- [17] Facchini, G., Favier, B., Le Gal, P., Wang, M. & Le Bars, M. 2018 The linear instability of the stratified plane Couette flow. *J. Fluid Mech.* 853, 205-234. · [Zbl 1415.76225](#)
- [18] Fedorovich, E. & Shapiro, A. 2009 Structure of numerically simulated katabatic and anabatic flows along steep slopes. *Acta Geophys.* 57 (4), 981-1010.
- [19] Fedorovich, E. & Shapiro, A. 2017 Oscillations in Prandtl slope flow started from rest. *Q. J. R. Meteorol. Soc.* 143 (703), 670-677.
- [20] Fernando, H. J. S., Pardyjak, E. R., Di Sabatino, S., Chow, F. K., De Wekker, S. F. J., Hoch, S. W., Hacker, J., Pace, J. C., Pratt, T., Pu, Z. et al. 2015 The MATERHORN: unraveling the intricacies of mountain weather. *Am. Meteorol. Soc. B* 96 (11), 1945-1967.
- [21] Fernando, H. J. S. & Weil, J. C. 2010 Whither the stable boundary layer? A shift in the research agenda. *Am. Meteorol. Soc. B* 91 (11), 1475-1484.
- [22] Giometto, M. G., Katul, G. G., Fang, J. & Parlange, M. B. 2017 Direct numerical simulation of turbulent slope flows up to Grashof number  $Gr = 2.1 \times 10^{11}$ . *J. Fluid Mech.* 829, 589-620. · [Zbl 1460.86026](#)
- [23] Görtler, H. 1959 Über eine analogie zwischen den instabilitäten laminarer grenzschichtströmungen an konkaven wänden und an erwärmten wänden. *Ing.-Arch.* 28 (1), 71-78. · [Zbl 0087.41201](#)
- [24] Grisogono, B. & Oerlemans, J. 2001a Katabatic flow: analytic solution for gradually varying eddy diffusivities. *J. Atmos. Sci.* 58 (21), 3349-3354.
- [25] Grisogono, B. & Oerlemans, J. 2001b A theory for the estimation of surface fluxes in simple katabatic flows. *Q. J. R. Meteorol. Soc.* 127 (578), 2725-2739.
- [26] Haaland, S. E. & Sparrow, E. M. 1973 Vortex instability of natural convection flow on inclined surfaces. *Intl J. Heat Mass Transfer* 16 (12), 2355-2367. · [Zbl 0275.76018](#)
- [27] Iyer, P. A. & Kelly, R. E. 1974 The stability of the laminar free convection flow induced by a heated inclined plate. *Intl J. Heat Mass Transfer* 17 (4), 517-525.
- [28] Jacobsen, D. A. & Senocak, I. 2013 Multi-level parallelism for incompressible flow computations on GPU clusters. *Parallel Comput.* 39 (1), 1-20.
- [29] Kosović, B. & Curry, J. A. 2000 A large eddy simulation study of a quasi-steady, stably stratified atmospheric boundary layer. *J. Atmos. Sci.* 57 (8), 1052-1068.
- [30] Kundu, P. K., Cohen, I. M. & Dowling, D. R. 2016 *Fluid Mechanics*, 6th edn. Elsevier. · [Zbl 1315.76002](#)
- [31] Le Dizès, S. & Billant, P. 2009 Radiative instability in stratified vortices. *Phys. Fluids* 21 (9), 096602.
- [32] Lin, M.-H. 2001 Numerical study of formation of longitudinal vortices in natural convection flow over horizontal and inclined surfaces. *Intl J. Heat Mass Transfer* 44 (9), 1759-1766. · [Zbl 1008.76083](#)
- [33] Lloyd, J. R. & Sparrow, E. M. 1970 On the instability of natural convection flow on inclined plates. *J. Fluid Mech.* 42 (3), 465-470.
- [34] Mahrt, L. 1998 Stratified atmospheric boundary layers and breakdown of models. *Theor. Comput. Fluid Dyn.* 11 (3-4), 263-279. · [Zbl 0948.76029](#)
- [35] Mahrt, L. 2014 Stably stratified atmospheric boundary layers. *Annu. Rev. Fluid Mech.* 46, 23-45. · [Zbl 1297.76077](#)
- [36] Mason, P. J. & Derbyshire, S. H. 1990 Large-eddy simulation of the stably-stratified atmospheric boundary layer. *Boundary-Layer Meteorol.* 53 (1-2), 117-162.
- [37] Miles, J. W. 1961 On the stability of heterogeneous shear flows. *J. Fluid Mech.* 10 (4), 496-508. · [Zbl 0101.43002](#)
- [38] Monin, A. S. & Obukhov, A. M. 1954 Basic laws of turbulent mixing in the atmosphere near the ground. *Tr. Akad. Nauk SSSR Geofiz. Inst* 24 (151), 163-187.
- [39] Monti, P., Fernando, H. J. S., Princevac, M., Chan, W. C., Kowalewski, T. A. & Pardyjak, E. R. 2002 Observations of flow and turbulence in the nocturnal boundary layer over a slope. *J. Atmos. Sci.* 59 (17), 2513-2534.
- [40] Nieuwstadt, F. T. M. 1984 The turbulent structure of the stable, nocturnal boundary layer. *J. Atmos. Sci.* 41 (14), 2202-2216.
- [41] Pera, L. & Gebhart, B. 1973 Natural convection boundary layer flow over horizontal and slightly inclined surfaces. *Intl J. Heat*

Mass Transfer 16 (6), 1131-1146. · [Zbl 0258.76034](#)

- [42] Prandtl, L. 1942 *Führer durch die Strömungslehre*. Vieweg und Sohn.
- [43] Prandtl, L. 1952 *Essentials of Fluid Dynamics: With Applications to Hydraulics, Aeronautics, Meteorology and other Subjects*. Blackie & Son. · [Zbl 0048.42801](#)
- [44] Salehipour, H., Caulfield, C. P. & Peltier, W. R. 2016 Turbulent mixing due to the Holmboe wave instability at high Reynolds number. *J. Fluid Mech.* 803, 591-621.
- [45] Sandu, I., Beljaars, A., Bechtold, P., Mauritsen, T. & Balsamo, G. 2013 Why is it so difficult to represent stably stratified conditions in numerical weather prediction (NWP) models? *J. Adv. Model. Earth Sy.* 5 (2), 117-133.
- [46] Schmid, P. J. & Henningson, D. S. 2001 *Stability and Transition in Shear Flows*. Springer. · [Zbl 0966.76003](#)
- [47] Schumann, U. 1990 Large-eddy simulation of the up-slope boundary layer. *Q. J. R. Meteorol. Soc.* 116 (493), 637-670.
- [48] Serafin, S., Adler, B., Cuxart, J., De Wekker, S. F. J., Gohm, A., Grisogono, B., Kalthoff, N., Kirshbaum, D. J., Rotach, M. W., Schmidli, J. et al. 2018 Exchange processes in the atmospheric boundary layer over mountainous terrain. *Atmosphere* 9 (3), 102.
- [49] Shah, S. K. & Bou-Zeid, E. 2014 Direct numerical simulations of turbulent Ekman layers with increasing static stability: modifications to the bulk structure and second-order statistics. *J. Fluid Mech.* 760, 494-539.
- [50] Shakespeare, C. J. 2019 Spontaneous generation of internal waves. *Phys. Today* 72 (6), 34-39.
- [51] Shakespeare, C. J. & Taylor, J. R. 2014 The spontaneous generation of inertia-gravity waves during frontogenesis forced by large strain: theory. *J. Fluid Mech.* 757, 817-853. · [Zbl 1416.76337](#)
- [52] Shapiro, A. & Fedorovich, E. 2004 Unsteady convectively driven flow along a vertical plate immersed in a stably stratified fluid. *J. Fluid Mech.* 498, 333-352. · [Zbl 1134.76463](#)
- [53] Sparrow, E. M. & Husar, R. B. 1969 Longitudinal vortices in natural convection flow on inclined plates. *J. Fluid Mech.* 37 (2), 251-255.
- [54] Steeneveld, G.-J. 2014 Current challenges in understanding and forecasting stable boundary layers over land and ice. *Front. Env. Sci. Engng* 2, 41.
- [55] Taylor, G. I. 1923 Stability of a viscous liquid contained between two rotating cylinders. *Phil. Trans. R. Soc. Lond.* A223 (605-615), 289-343.
- [56] Turner, J. S. 1979 *Buoyancy Effects in Fluids*. Cambridge University Press. · [Zbl 0443.76091](#)
- [57] Umphrey, C., DeLeon, R. & Senocak, I. 2017 Direct numerical simulation of turbulent katabatic slope flows with an immersed boundary method. *Boundary-Layer Meteorol.* 164 (3), 367-382.
- [58] Whiteman, C. D. 1990 Observations of thermally developed wind systems in mountainous terrain. In *Atmospheric Processes Over Complex Terrain*, pp. 5-42. Springer.
- [59] Whiteman, C. D. 2000 *Mountain Meteorology: Fundamentals and Applications*. Oxford University Press.
- [60] Xiao, C. & Senocak, I. 2019 Stability of the Prandtl model for katabatic slope flows. *J. Fluid Mech.* 865, R2.
- [61] Zardi, D. & Whiteman, C. D. 2013 Diurnal mountain wind systems. In *Mountain Weather Research and Forecasting*, pp. 35-119. Springer.

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