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A noncommutative cycle index and new bases of quasi-symmetric functions and noncommutative symmetric functions. (English) Zbl 1460.16037

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Summary: We define a new basis of the algebra of quasi-symmetric functions by lifting the cycle-index polynomials of symmetric groups to noncommutative polynomials with coefficients in the algebra of free quasi-symmetric functions, and then projecting the coefficients to QSym. By duality, we obtain a basis of noncommutative symmetric functions, for which a product formula and a recurrence in the form of a combinatorial complex are obtained. This basis allows to identify noncommutative symmetric functions with the quotient of FQSym induced by the pattern-replacement relation $321 \equiv 231$ and $312 \equiv 132$.

MSC:

16T30 Connections of Hopf algebras with combinatorics
05E05 Symmetric functions and generalizations
05A18 Partitions of sets

Keywords:

noncommutative symmetric functions; quasi-symmetric functions; dendriform algebras

Software:

OEIS

Full Text: DOI

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