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Spectral analysis of model Couette flows in application to the ocean. (English. Russian original)

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Summary: A method for analysis of the evolution equation of potential vorticity in the quasi-geostrophic approximation with allowance for vertical diffusion of mass and momentum for analyzing the stability of small perturbations of ocean currents with a linear vertical profile of the main flow is developed. The problem depends on several dimensionless parameters and reduces to solving a spectral non-self-adjoint problem containing a small parameter multiplying the highest derivative. A specific feature of this problem is that the spectral parameter enters into both the equation and the boundary conditions. Depending on the types of the boundary conditions, problems I and II, differing in specifying either a perturbations of pressure or its second derivative, are studied. Asymptotic expansions of the eigenfunctions and eigenvalues for small wavenumbers k are found. It is found that, in problem I, as $k \to +0$, there are two finite eigenvalues and a countable set of unlimitedly increasing eigenvalues lying on the line $\mathrm{Re}(c) = \frac{1}{2}$. In problem II, as $k \to +0$, there are only unlimitedly increasing eigenvalues. A high-precision analytical-numerical method for calculating the eigenfunctions and eigenvalues of both problems for a wide range of physical parameters and wavenumbers k is developed. It is shown that, with variation in the wavenumber k, some pairs of eigenvalues form double eigenvalues, which, with increasing k, split into simple eigenvalues, symmetric with respect to the line $\mathrm{Re}(c) = \frac{1}{2}$. A large number of simple and double eigenvalues are calculated with high accuracy, and the trajectories of eigenvalues with variation in k, as well as the dependence of the flow instability on the problem parameters, are analyzed.

MSC:

76E20	Stability and instability of geophysical and astrophysical flows
76M22	Spectral methods applied to problems in fluid mechanics
76M45	Asymptotic methods, singular perturbations applied to problems in fluid mechanics
86A05	Hydrology, hydrography, oceanography

Keywords:

spectral non-self-adjoint problem; system Wronskian; Newton method; asymptotic expansion; double eigenvalue

Full Text: DOI

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