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Mixed convolved action for the fractional derivative Kelvin-Voigt model. (English)

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Summary: Based upon the concept of mixed convolved action, a true variational statement for a fractional derivative Kelvin-Voigt model is presented. In this formulation, a single functional is defined as a series of convolution integrals, where the stationarity of this functional leads to all the governing differential equations as well as pertinent initial conditions. Thus, the entire description of a fractional-derivative Kelvin-Voigt model is encapsulated within this framework. This new formulation provides an elegant basis for a development of effective numerical methods involving finite element representation over a temporal domain. Here, the simplest temporal finite element approach is developed, and some computational examples are provided to validate this proposed approach.

MSC:

74D10 Nonlinear constitutive equations for materials with memory
74S40 Applications of fractional calculus in solid mechanics

Software:

Mathematica; Maple

Full Text: DOI

References:

[1] Sivaselvan, MV; Reinhorn, AM, Lagrangian approach to structural collapse simulation, J. Eng. Mech. ASCE, 132, 795-805 (2006)

[2] Lavan, O.; Sivaselvan, MV; Reinhorn, AM; Dargush, GF, Progressive collapse analysis through strength degradation and fracture in the Mixed Lagrangian Formulation, Earthq. Eng. Struct. D, 38, 1483-1504 (2009)

[3] Sivaselvan, MV; Lavan, O.; Dargush, GF; Kurino, H.; Hyodo, Y.; Fukuda, R.; Sato, K.; Apostolakis, G.; Reinhorn, AM, Numerical collapse simulation of large-scale structural systems using an optimization-based algorithm, Earthq. Eng. Struct. D, 38, 655-677 (2009)

[4] Apostolakis, G.; Dargush, GF, Mixed Lagrangian formulation for linear thermoelastic response of structures, J. Eng. Mech. ASCE, 138, 508-518 (2012)

[5] Apostolakis, G.; Dargush, GF, Mixed variational principles for dynamic response of thermoelastic and poroelastic continua, Int. J. Solids Struct., 50, 642-650 (2013)

[6] Apostolakis, G.; Dargush, GF, Variational methods in irreversible thermoelasticity: theoretical developments and minimum principles for the discrete form, ACTA Mech., 224, 2065-2088 (2013) · Zbl 1398.74062

[7] Kim, J.; Dargush, GF; Ju, YK, Extended framework of Hamilton's principle for continuum dynamics, Int. J. Solids Struct., 50, 3418-3429 (2013)

[8] Oldham, KB; Spanier, J., The Fractional Calculus: Theory and Applications of Differentiation and Integration to Arbitrary Order (1974), New York: Academic Press, New York · Zbl 0292.26011

[9] Samko, SG; Kilbas, AA; Marichev, OI, Fractional Integrals and Derivatives: Theory and Applications (1993), Philadelphia: Gordon and Breach Science Publishers, Philadelphia · Zbl 0818.26003

[10] Gurtin, ME, Variational principles for linear elastodynamics, Arch. Ration. Mech. Anal., 16, 34-50 (1964) · Zbl 0124.40001

[11] Gurtin, ME, Variational principles for linear initial-value problems, Q. Appl. Math., 22, 252-256 (1964) · Zbl 0173.37602

[12] Tonti, E., On the variational formulation for linear initial value problems, Ann. Mat. Pur. Appl., 95, 331-359 (1973) · Zbl 0278.49047

[13] Dargush, GF, Mixed convolved action for classical and fractional-derivative dissipative dynamical systems, Phys. Rev. E, 86, 066606 (2012)

[14] Dargush, GF; Kim, J., Mixed convolved action, Phys. Rev. E, 85, 066606 (2012)

[15] Dargush, GF; Darrall, BT; Kim, J.; Apostolakis, G., Mixed convolved action principles in linear continuum dynamics, ACTA Mech., 226, 4111-4137 (2015) · Zbl 1336.74013

[16] Dargush, GF; Apostolakis, G.; Darrall, B.; Kim, J., Mixed convolved action variational principles in heat diffusion, Int. J. Heat Mass Tran., 100, 790-799 (2016) · Zbl 1336.74013

[17] Kim, J.; Shin, J., Mixed convolved action for thermoelasticity, Int. J. Appl. Mech., 10, 1850002 (2018)

[18] Darrall, BT; Dargush, GF, Variational principle and time-space finite element method for dynamic thermoelasticity based on mixed convolved action, Eur. J. Mech. A-Solid, 71, 351-364 (2018) · Zbl 1406.74175

[19] Hamilton, WR, On a general method in dynamics, Philos. Trans. R. Soc. Lond., 124, 247-308 (1834)

[20] Hamilton, WR, Second essay on a general method in dynamics, Philos. Trans. R. Soc. Lond., 125, 95-144 (1835)

[21] Rayleigh, JWS, The Theory of Sound (1877), London: Macmillan, London · Zbl 15.0848.02

[22] Schiessel, H.; Metzler, R.; Blumen, A.; Nonnenmacher, TF, Generalized viscoelastic models: their fractional equations with solutions, J. Phys. A-Math. Gen., 28, 6567-6584 (1995) · Zbl 0921.73154

[23] Rossikhin, YA; Shitikova, MV, Application of fractional calculus for dynamic problems of solid mechanics: novel trends and recent results, Appl. Mech. Rev., 63, 010801 (2010)

[24] Ingman, D.; Suzdalnitsky, J., Response of viscoelastic plate to impact, J. Vib. Acoust., 130, 011010 (2008)

[25] Katsikadelis, JT, Nonlinear dynamic analysis of viscoelastic membranes described with fractional differential models, J. Theor. Appl. Mech., 50, 743-753 (2012)

[26] Di Paola, M.; Heuer, R.; Pirrotta, A., Fractional visco-elastic Euler-Bernoulli beam, Int. J. Solids Struct., 50, 3505-3510 (2013)

[27] Freundlich, J., Vibrations of a simply supported beam with a fractional viscoelastic material model-supports movement excitation, Shock Vib., 20, 1103-1112 (2013)

[28] Demir, DD; Bildik, N.; Sınır, B., Linear dynamical analysis of fractionally damped beams and rods, J. Eng. Math., 85, 131-147 (2014) · Zbl 1359.74204

[29] Cajic, M.; Karlicic, D.; Lazarevic, M., Nonlocal vibration of a fractional order viscoelastic nanobeam with attached nanoparticle, Theor. Appl. Mech., 42, 167-190 (2015) · Zbl 1462.74061

[30] Rossikhin, YA; Shitikova, M., The fractional derivative Kelvin-Voigt model of viscoelasticity with and without volumetric relaxation, J. Phys. Conf. Ser., 991, 012069 (2018)

[31] Achar, BNN; Hanneken, JW; Enck, T.; Clarke, T., Dynamics of the fractional oscillator, Phys. A, 297, 361-367 (2001) · Zbl 0969.70511

[32] Stanislavsky, AA, Fractional oscillator, Phys. Rev. E, 70, 051103 (2004)

[33] Martinez-Agirre, M.; Elejabarrieta, MJ, Characterisation and modelling of viscoelastically damped sandwich structures, Int. J. Mech. Sci., 52, 1225-1233 (2010)

[34] Song, XY; Cao, TN; Gao, PX; Han, QK, Vibration and damping analysis of cylindrical shell treated with viscoelastic damping materials under elastic boundary conditions via a unified Rayleigh-Ritz method, Int. J. Mech. Sci., 165, 105158 (2020)

[35] Love, ER; Young, L., On fractional integration by parts, Proc. Lond. Math. Soc., 2, 1-35 (1938) · Zbl 64.0197.02

[36] Caputo, M., Linear models of dissipation whose Q is almost frequency independent-II, Geophys. J. Int., 13, 529-539 (1967)

[37] Kilbas, AA; Srivastava, HM; Trujillo, JJ, Theory and Applications of Fractional Differential Equations (2006), New York: Elsevier, New York · Zbl 1092.45003

[38] Abdeljawad, T., On Riemann and Caputo fractional differences, Comput. Math. Appl., 62, 1602-1611 (2011) · Zbl 1228.26008

[39] Diethelm, K., The Analysis of Fractional Differential Equations: An Application-Oriented Exposition Using Differential Operators of Caputo Type (2010), Berlin: Springer, Berlin · Zbl 1215.34001

[40] Caputo, M., Elasticita e dissipazione (1969), Bologna: Zanichelli, Bologna

[41] Torvik, PJ; Bagley, RL, On the appearance of the fractional derivative in the behavior of real materials, J. Appl. Mech. Trans. ASME, 51, 294-298 (1984) · Zbl 1203.74022

[42] Beyer, H.; Kempfle, S., Definition of physically consistent damping laws with fractional derivatives, Z. Angew. Math. Mech., 75, 623-635 (1995) · Zbl 0865.70014

[43] Podlubny, I., Fractional Differential Equations: An Introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of Their Solution and Some of Their Applications (1999), Sandiego: Academic Press, Sandiego · Zbl 0924.34008

[44] Galucio, AC; Deü, J-F; Mengué, S.; Dubois, F., An adaptation of the Gear scheme for fractional derivatives, Comput. Method Appl. Mech., 195, 6073-6085 (2006) · Zbl 1123.65123

[45] Deü, J-F; Matignon, D., Simulation of fractionally damped mechanical systems by means of a Newmark-diffusive scheme, Comput. Math. Appl., 59, 1745-1753 (2010) · Zbl 1189.65137

[46] Maplesoft, MAPLE, Canada (2009)

[47] W.R. Inc, Mathematica. Champaign, Illinois (2019)

[48] Benguria, R.; Depassier, M., Variational principle for limit cycles of the Rayleigh-van der Pol equation, Phys. Rev. E, 59, 4889-4893 (1999)

[49] Liu, Y.; Mojahed, A.; Bergman, LA; Vakakis, AF, A new way to introduce geometrically nonlinear stiffness and damping with an application to vibration suppression, Nonlinear Dyn., 96, 1819-1845 (2019) · Zbl 1437.70034