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A robust high-order discontinuous Galerkin method with large time steps for the compressible Euler equations. (English) Zbl 1457.65134

Commun. Math. Sci. 15, No. 3, 813-837 (2017).

Summary: We present a high-order Lagrange-projection like method for the approximation of the compressible Euler equations with a general equation of state. We extend the method introduced in [the author, Numer. Math. 135, No. 2, 493–519 (2017; Zbl 1381.76198)] in the case of the isentropic gas dynamics to the compressible Euler equations and minimize the numerical dissipation by quantifying it from a parameter evaluated locally in each element of the mesh. The method is based on a decomposition between acoustic and transport operators associated to an implicit-explicit time integration, thus relaxing the constraint of acoustic waves on the time step as proposed in [*F. Coquel* et al., Math. Comput. 79, No. 271, 1493–1533 (2010; Zbl 1369.76032)] in the context of a first-order finite volume method. We derive conditions on the time step and on a local numerical dissipation parameter to keep positivity of the mean value of the discrete density and internal energy in each element of the mesh and to satisfy a discrete inequality for the physical entropy at any approximation order in space. These results are then used to design limiting procedures in order to restore these properties at nodal values within elements. Moreover, the scheme is designed to avoid over-resolution in space and time in the low Mach number regime. Numerical experiments support the conclusions of the analysis and highlight stability and robustness of the present method when applied to either discontinuous flows or vacuum. Large time steps are allowed while keeping accuracy on smooth solutions even for low Mach number flows.

MSC:

65M60	Finite element, Rayleigh-Ritz and Galerkin methods for initial value and initial-boundary value problems involving PDEs	Cited in **3** Documents
65M06	Finite difference methods for initial value and initial-boundary value problems involving PDEs	
65M12	Stability and convergence of numerical methods for initial value and initial-boundary value problems involving PDEs	
76N06	Compressible Navier-Stokes equations	
76N15	Gas dynamics (general theory)	
76H05	Transonic flows	
76M10	Finite element methods applied to problems in fluid mechanics	
35Q31	Euler equations	

Keywords:

Lagrange-projection; discontinuous Galerkin method; explicit-implicit; entropy-satisfying; positivity-preserving; compressible Euler equations

Software:

Coq

Full Text: DOI