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On the Hopf (double Hopf) bifurcations and transitions of two-layer western boundary currents. (English) Zbl 1456.76055

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Summary: This study examines the instability and dynamical transitions of the two-layer western boundary currents represented by the Munk profile in the upper layer and a motionless bottom layer in a closed rectangular domain. First, a bound on the intensity of the Munk profile below which the western boundary currents are locally nonlinearly stable is provided. Second, by reducing the infinite dimensional system to a finite dimensional one via the center manifold reduction, non-dimensional transition numbers are derived, which determine the types of dynamical transitions both from a pair of simple complex eigenvalues as well as from a double pair of complex conjugate eigenvalues as the Reynolds number crosses a critical threshold. We show by careful numerical estimations of the transition numbers that the transitions in both cases are continuous at the critical Reynolds number. After the transition from a pair of simple complex eigenvalue, the western boundary layer currents turn into a periodic circulation, whereas a quasi-periodic or possibly a chaotic circulation emerges after the transition from a pair of double complex eigenvalues. Finally, a comparison between the transitions exhibited in one-layer and two-layer models is provided, which demonstrates the fundamental differences between the two models.

MSC:

76E20 Stability and instability of geophysical and astrophysical flows
86A05 Hydrology, hydrography, oceanography

Keywords:

dynamic transition; continuous transition; catastrophic transition; Munk profile

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