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Bracketing backward reach sets of a dynamical system. (English) Zbl 1454.93023
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Summary: In this paper, we present a new method for bracketing (i.e. characterising from inside and from outside) backward reach set of the target region \mathbb{T} of a continuous-time dynamical system. The principle of the method is to formalise the problem as a *constraint network*, where the variables are the trajectories (or paths) of the system. The resolution is made possible by using *mazes* which is a set of paths that contain all solutions of the problem. As a result, we will be able to derive a method able to compute a backward reach set for a huge class of systems without any knowledge of a parametric Lyapunov function and without assuming any linearity for our system. The method will be illustrated in several examples.

MSC:

93B03 Attainable sets, reachability
93B70 Networked control

Keywords:

abstract interpretation; backward reach set; basin of attraction; stability; constraint solving; interval computation

Software:

GloptiPoly; Acumen; AQCS

Full Text: [DOI](#)

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