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Conti, Sergio; Dolzmann, Georg

Optimal laminates in single-slip elastoplasticity. (English) Zbl 1454.74021

Discrete Contin. Dyn. Syst., Ser. S 14, No. 1, 1-16 (2021).

Summary: Recent progress in the mathematical analysis of variational models for the plastic deformation of crystals in a geometrically nonlinear setting is discussed. The focus lies on the first time-step and on situations where only one slip system is active, in two spatial dimensions. The interplay of invariance under finite rotations and plastic deformation leads to the emergence of microstructures, which can be analyzed in the framework of relaxation theory using the theory of quasiconvexity. A class of elastoplastic energies with one active slip system that converge asymptotically to a model with rigid elasticity is presented and the interplay between relaxation and asymptotics is investigated.

MSC:

74C15 Large-strain, rate-independent theories of plasticity (including nonlinear plasticity)
74B20 Nonlinear elasticity
49J45 Methods involving semicontinuity and convergence; relaxation

Keywords:

elastoplasticity; single slip; quasiconvexity; relaxation; optimal laminates

Full Text: DOI

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